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SPECTRAL ANALYSIS OF NON-STATIONARY TIME SERIES

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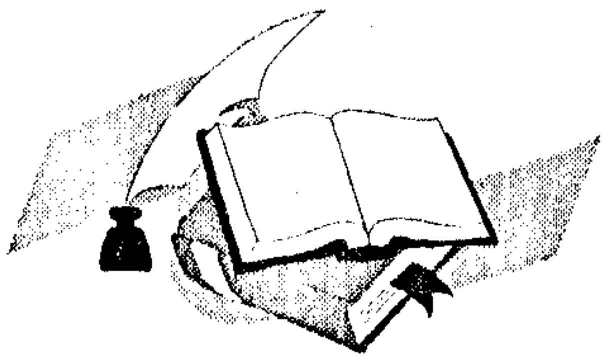

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1. Introduction;

The aim of this paper is to take stock of the important recent contributions to spectral analysis, especially as they apply to non-stationary processes. Non-stationary processes are particularly relevant in the empirical sciences where most phenomena exhibit pronounced departures from stationary. That detrending and various other filtering operations to induce stationarity lead to distortions in the true spectrum has been known for quite some time now (see Slutzky (1937), Moran (1953), Grenander and Rosenblatt (1957) etc.). This suggests that analysing the spectrum of a non-stationary process directly may have much to recommend itself.

The plan of the paper is as follows. Section 2 is devoted to preliminaries. Some early attempts to introduce time-changing spectra are analysed in section 3. The important concept of "evolutionary spectrum" due to Priestley (1988) is discussed in section 4, whereas **the** fundamental contributions of Zurbenko (1986) are reviewed in section 5 Conclusions are gathered in section 6.

2. **Preliminaries;**

Since several of the results mentioned in section 5 depend on "mixing" conditions, we begin by defining 2 types of mixing conditions.

Rosenblatt mixing condition (Rosenblatt (1985)} Let X(t) be a time-series and let Π_a^b denote the a-algebra generated by the random variables $X(t),\ t \in [a,b]$. The Rosenblatt mixing condition states that

$$\sup_t \left| p(AB) - p(A)p(B) \right| \leq \alpha_n \qquad (1)$$

where $A \in \Pi_{-\infty}^{t}$, $B \in \Pi_{t+n}^{\infty}$

Ibragimov mixing condition (Ibragimov(1962));

$$\sup_t \left| p(A/B) - p(A) \right| \leq \beta_n \qquad (2)$$

where A,B are as defined above.

Stated in this form, the Ibragimov mixing condition is not satisfied by stochastic processes with values in Hilbert or Banach spaces, 'and hence, the additional restriction.

$$\sum_{n=-\infty}^{\infty} \beta_{|n|}^{1/2} = \beta < \infty \qquad (2)$$

is usually imposed.

For a discrete parameter stationary process X_t we have the following spectral representation.

$$X_t = \int_{-\Pi}^{\Pi} \exp(it\omega) Z(d\omega) \qquad (3)$$

where B(w) is a stochastic process with orthogonal increments.

Let $S_n\ (t_1 .. t_n),\ n \geq 1$ denote the cumulanta of the stationary process X_t - they are shift-invariant because of stationarity.

Spectral measures :the quantity F_n defined on the cube Π^n = [-n,nj x .... x f-n,nj (n times) by

$$S_n\ (t_1 ... t_n) = \int_{\Pi^n} \exp(i \sum_{k=1}^{n} t_k \omega_k)\ F_n(d\bar{\omega}) \qquad (4)$$

where $\bar{\omega} = (\omega_1 ... \omega_n)$ is called the spectral measure of order n.

It can be shown that for a stationary process **the** spectral measures F_n are concentrated on the manifolds $\omega_1 + ... \omega_n = 0$ (mod 2Π) and can be written in the form

$$F_n(M) = \int_M f_n(\omega_1, \quad \omega_n)\ \delta^*(\omega_1 + ... \omega_n) d\bar{\omega} \quad . \qquad (5)$$

where? $M \subset \Pi^n$ and $\delta^*(x) = \sum_t \delta(x-2\Pi t)$, $\delta(x)$ being the Dirac delta-function. The quantities f_n may be called the spectral densities (n*2 corresponds to the ordinary spectrum, n=3 to the bispectrum etc.) (See e.g. Brillinger (1975))

3. Time-dependent spectra:

Page's spectrum

Possibly the first attempt to define a time-dependent a spectrum occurs in Page (1952). For a continuous parameter process {X (t)), Page introduces the quantity

$$g_T^*(\omega) = \left| \int_0^T X(t) \exp(-i\omega t)dt \right|^2 \quad (6)$$

and defines the instantaneous power spectrum $f_t(\omega)$ as

$$f_t(\omega) = (d/dt)\ E(g_t^*(\omega)) \quad (7)$$

Thus f_t (w) roughly measures the difference between the power distribution of the process over the interval <o,t) and over the interval (o,t + dt).

Mark's Physical Spectrum

For a continuous parameter process X(t), Hark (1970) introduces the concept of the physical spectrum as follows!

$$S(\omega,t,W) = E\left[\left| \int_{-\infty}^{\infty} W(t-u)X(u) \exp(-i\omega u)du \right|\right]^2 \quad (8)$$

where W(t> is a suitable real-valued function with W(0) > 0 ,W(t> is concentrated in the neighborhood of t = 0 and

$$\int_{-\infty}^{\infty} W^2(t)dt = 1$$

Tjostheim Spectrum;

Cramer (196JL) has shown that for a discrete parameter process X_t which is purely non-deterministic, the following I "-aided linear representation exists

$$X_t = \sum_{u=0}^{\infty} a_t(u)\varepsilon_{t-u} \quad (9)$$

Where ε_t is a white noise innovation. Tjostheim (1976) proposed a definition of a time-dependent spectrum- based on (9)

$$f_t(\omega) = (\sigma_\varepsilon^2 \;/2\Pi)\left|\sum_{u=0}^{\infty} a_t(u) \quad \exp(-i\omega u)\right|^2 \quad (10)$$

where $\sigma_\varepsilon^2 = \mathrm{var}(\varepsilon_t)$.

Melard (1978, 1985) has suggested a similar approach.

Details of these early attempts as well as some of their limitations have been reviewed in Priestley (1988), where also the concept of an evolutionary spectrum is introduced.

4. Priestley'a Evolutionary Spectrum:

Priestley developed the concept of the evolutionary spectrum in a series of papers (Priestley (1965, 1966, 1969) but finds its clearest exposition in Priestley (1988). In the interests of uniformity, throughout this discussion, the underlying process X(t) is assumed to be a complex continuous parameter process* If X(t) were stationary, the representation (3) would be possible and then the covariance kernel R(s,t) Would admit the corresponding representation

$$R(s,t) = \int_{-\infty}^{\infty} \exp(i\omega t - i\omega s) \; dH(\omega) \quad (11)$$

where H(w) is the integrated spectrum of X(t).

For non-stationary processes both representations (3) and (11) are ruled out. However as shown by Priestley (1981) an appeal to the theory of "general orthogonal expansions" can yield for a fairly general class of stochastic processes the following representation for R(s,t)

$$R(s,t) = \int_{-\infty}^{\infty} \phi_s(\omega)\phi_t(\omega) \; d\mu(\omega) \qquad (12)$$

where $\{\phi_t(w)\}$ are a family of functions F defined on the real line and $\mu(w)$ is a measure on the real line. Sharper results might, be obtained by assuming that $\mu(\omega)$ is absolutely, continuous w.r.t. the Lebesgue measure on the real line.

Of special significance are the so-called oscillatory functions $\phi_t(w)$..

Oscillatory function: the function $\phi_t(w)$ will be called an Oscillatory function if for some θ(w) it can be written in the form

$$\phi_t(w) = A_t(\omega)\ \exp(i\theta(\omega)t) \qquad (13)$$

$$A_t(w) = \int^{\infty} \exp(it\theta)\ dK_{\omega}(\theta) \qquad (14)$$

where θ=*0* yields an absolute maximum for $|dK_{\omega}(\theta)|$

Oscillatory Process if there exists a family of oscillatory functions $\{\phi_t(w)\}$ in terms of which X (t) has the representation

$$X(t) = \int_{-\infty}^{\infty} \phi_t(\omega)\ dZ(\omega) \qquad (15)$$

with Z(w) an orthogonal process with

$$E\ [\,|dZ(\omega)|^2\,] = d\mu(\omega) \qquad (16)$$

then the process X<t) will be termed an oscillatory process.

For any particular process {X(t)l there would exist in genera] a number of different families of oscillatory functions in terms of each of which $\{X_t\}$ has the representation (1.5). If F denotes a specific family of such oscillatory functions (of the form (13), then the evolutionary power Spectrum at time t w.r.t. F is defined by Priestley as dH t (w) where

$$dH^*_t(\omega) = |A_t(\omega)|^2\ d\mu(\omega) \qquad (17)$$

when additionally the measure $\mu(\omega)$ is absolutely continuous w.r.t. the Lebesgue measure we may write for each t

$$dH_t(\omega) = h_t(\omega)d\omega \qquad (18)$$

and $h_t(\omega)$ may then be called the evolutionary spectral densıty ıunction

Bandwidth Considerations For each family F with oscillatory functions represented by (13), the following quantity is defined

$$B_F(w) = \int|\theta|\,|dK_{\omega}(\theta)| \qquad (19)$$

define B↓F as

$$B_F = \sup_{\omega}\ \{B_F(\omega)\}^{-1} \qquad (20)$$

If B_F is finite, the family P is called semi-stationary and B_F itself is called as the characteristic width of the family P.
A semi-stationary process {X(t)l is one for which 3 a semi-stationary family Fwhich can furnish a spectral representation for X(t).
Let C denote the class of all such semi-stationary families.
Define B_X by

$$B_X = \sup_{F \in \mathbb{C}} \{B_F\} \quad (21)$$

then B_X termed the characteristic width of the semi-stationary process {X(t)}.

Estimation; Let X(t) be a continuous parameter semi-stationary process over (O,T), with measures corresponding to the semi-stationary families, absolutely continuous (w.r.t. the Lebesgue measure). The evolutionary spectral density h_t may then be estimated by the 2-atage procedure suggested by# Priestley and Tong (1973) which involves (i) first passing the data through a linear filter concentrated on a typical frequency w_o y and yielding an output $u(t)$ (ii) and second, computing a weighted average of $|u(t)|^2$ in the neighborhood of the time point t. This estimate Formally expressed

$$u(t) = \int_{t-T}^{t} g(u)\ X(t-u)\ \exp(-i\omega_o(t-u))\ d\mu \quad (22)$$

where g(u) is *a* filter whose transfer function r(a>) is peaked in the neighborhood of w = o and is normalised so that its square integrates to 1 over the range $(-\infty, \infty)$. lter width B_g is much smaller than B_X› achieve high time domain resolution). The

total power density at ω_o may then be defined as

$$\hat{h}_t(\omega_o) = \int_{t-T}^{t} \omega(v)|\ u(t-v)|^2\ dv \quad (23)$$

where w(v) is normalised to integrate to unity and has a "width" substantially in excess of B_g (for attaining high frequency-domain resolution);

The extension to the discrete parameter case Straight forward and an economic application may be found in Nachane and Ray (forthcoming 1992)*

5. Kolmogorov-Zurhenko Results:

A The Stationary Case:

Several results of fundamental significance in spectra] analysis were initiated by Kolmogorov and Zurbenko (1978) and were later followed up by Zurbenko (1980, 1982, 1986). To view these results in perspective, a quick retreading of familiar grounds may be necessary.

Let X(t) be a stationary time series, on which the record {X(1),...X(N)} of length N is available. The spectrum of X(t) is denoted by f(M and it is assumed that E(X(t))=0.

The estimate $\hat{f}_N(\lambda)$ defined by

$$\hat{f}_N(\lambda) = \int_{-\pi}^{\pi} \phi_N(x)\, I_N(x+\lambda)\, dx \qquad (24)$$

(where $I_N(x)$ is the modified periodogram and $\phi_N(x)$ is a function continuous in $(-\pi,\pi)$ with Fourier coefficients $b^{(N)}(t)$) is called an estimate of the Grenander-Rosenblatt type.

Parzen (1957) focused attention on a more restricted class

of the estimates $\hat{f}_N(\lambda)$ in which the spectral window <£ (x) can be represented as

$$\phi_N(x) = A_N\, \phi(A_N\, x) \qquad 1 < A_N < N \qquad (25)$$

where

$$\phi(x) = (1/2\pi)\int_{-\infty}^{\infty} K(t)\exp(itx)\, dt \qquad (26)$$

and

$$K(t/A_N) = b^{(N)}(t) \qquad (27)$$

The function **K(x)**, as is well-known, is called the oovariance window of the estimate. Parzen (1957) then proved the following theorem, which bears his name.

Theorem: Suppose that for some **a** > 0 the quantity

$$\sum_{t=-\infty}^{\infty} |t|^{\alpha} c(t) < \infty$$

(where c(t) is the auto-covariance function) and further that the A_N in (25) are chosen to satisfy

$$\lim_{N \to \infty} \left(\frac{N^{1/(1+2\alpha)}}{A_N} \right) = a \qquad (29)$$

Let $K^{(r)}$ the index of K(x) i.e. r is the largest value of r for which

$$\lim_{x \to 0} \frac{1 - K(x)}{|x|^{r^*}} \qquad (30)$$

exists

Then $\lim_{N \to \infty} N^{2\alpha/(1+2\alpha)} \, MSE(\hat{f}_N(\lambda))$

$$= a^{-1} f^2(\lambda)(1+\eta(\lambda)) \int_{-\infty}^{\infty} K(x)dx + a^{2\alpha} |K^{(\alpha)} f^{(\alpha)}(\lambda)|^2 \qquad (31)$$

provided the following conditions hold.

(i) $r > \alpha$

(ii) the process X(t) possesses **moments** of **the 4th order.**

The function $\eta(\lambda)$ is defined as follows

$$\eta(\lambda) \begin{cases} = 1 & \text{if } \lambda \neq 0 \bmod(\pi) \\ = 0 & \text{if } \lambda = 0 \bmod(\pi) \end{cases} \qquad (32)$$

The above theorem implies that the least possible order of the MSE of $f_N(\lambda)$ is $N^{-2\alpha_o/(1+2\alpha_o)}$ where α_o the maximal value of ot for which (28) is true. However this may not be the least possible order on the class of **all** estimates $\hat{f}_N(\lambda)$ of the Grenander-Rosenblatt type. The solution of this more general problem was fully worked out by Zurbenko (1978) and Vorobjev and Zurbenko (1979). For the sake of expository simplicity the discussion is restricted to discrete parameter processes X_t

Let $\{X_o \ldots X_N\}$ be a record of such a process and choose
integers L, M, T (functions of N) st

(i) L < M < N
(ii) N = (T-l) T, + M + 1 (33)
and (iii) LT =N

Let $a_M(t)$ be a non-negative function vanishing outside [o,M].

Define

$$W_M^Q(\lambda) = \sum_{t=-\infty}^{\infty} a_M(t-Q)\ X(t)\ \exp(it\lambda) \qquad (34)$$

and $$\phi_M^Q(x) = \sum_{t=-\infty}^{\infty} a_M(t-Q)\ \exp(itx) \qquad (35)$$

It can be seen that

$$\phi_M^Q(x) = \phi_M^0(x)\ \exp(iQx) \qquad (36)$$

We now assume that the coefficients $a_M(t)$ are chosen such that

$$\int^{\pi} |\phi_M^0(x)|^2 dx = 1 \qquad (37)$$

Zurbenko then defines the estimate fN (λ) of the spectral density

f(λ) of X,. by

$$\tilde{f}_N(\lambda) = (1/T)\sum_{k=0}^{T-1} |W_M^{Lk}(\lambda)|^2$$

with L,k,T chosen to satisfy (33)

The asymptotic behaviour of the MSB of (38) is obtained in Zurbenko (1986), for a special class of processes X(t) characterised as follows!

Definition: The set O is defined as the set of stochastic processes with zero mean which possess second and fourth order spectral densities.

Definition: Suppose X(t) =O ; we now define a subset of O viz.

W (λ,f ,f $_1$ .. .f[α]. ., α, C, C_1) if V p and some given λ, f $\geq$ o,

$\alpha > 0$ C $\geq$ 0, C1$\geq$ 0 with the f1's as defined in (5) (Also

α) denotes the integral part of a) the following holds

(i) $\left| f(\lambda+\mu) - f - f_1\mu - f_2(\mu^2/2!) \ldots - f_{[\alpha]}(\mu^{[\alpha]}/[\alpha]!) \right| < C|\mu|^{\alpha}$

and

(ii) the 4th order spectral density $f_4(x_1 \ldots x_4)$ of X(t) is bounded

$$\left| f_4(x_1 \ldots x_4) \le C_1 \right| \qquad (40)$$

The spectral kernels (35) are also somewhat restricted. Specifically the following conditions are imposed.

(i) For any Q, the sequence of functions { $\phi_M^Q(x)$ has each member evencontinuous, and of period 2Π,

(ii) Each $\phi_M^Q(x)$ tends to o uniformly in the region e ≤ |x| ≤ Π for any ∈ > o.

(iii) $\int_{-\pi}^{\pi} \phi_M^Q(x)\, dx = 1$

$$\int_{-\pi}^{\pi} |\phi_M^Q(x)|\, dx = \| \phi_M^Q \|$$

and

$$\sup_M \| \phi_M^Q \| < \infty$$

and finally

(iv) If $X(t) \in W(\lambda, f, f_1 \ldots f_{[\alpha]}, \alpha, C, C_1)$

then

$$\sum_{k \in (1, L\pi/C)} \sup_{x \in [kC/L, \pi]} |\phi_M^Q(x)|^2 = O(L)$$

The lower bound (asymptotic) of the MSB is now given by the following theorem.

Theorem: For the estimate f N (λ) with λ fixed, we have

$$\lim_{N \to \infty} \inf_{\phi_M^Q(x)} \sup_{X(t)} (N^{2\alpha/(1+2\alpha)} MSE\, \bar{f}_N(\lambda))$$

$$\ge H(\alpha) f^2(\lambda) \, (C^2/f^2(\lambda))^{1/(1+2\alpha)} \, (1+\eta(\lambda))^{2\alpha/(1+2\alpha)}$$

where X(t) ∈ W ( ) and ^ $\phi_M^Q(x)$ (x)are restricted as defined earlier; n((λ) is defined by (32) and

In H(α) = -(l/l + 2α) ln(l + 2α) + (2α/l+2α) ln(n (α + 1)/ α) (42)

For the practical choice of $a_M(t)$, two alternatives may be considered, either a Bartlett (1950) type of window or a Kolmogorov-Zurbenko <1978) type of window.

$$\text{Bartlett: } a_M(t) = \begin{cases} (1/\sqrt{2\pi M}) & t \in [0, M-1] \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Kolmogorov-Zurbenko: } a_M(t) = \mu(K,P)\,[K(P^2-1)/12\pi]^{1/4} P^{-K} C_{K,P}(t)$$

where M =K <p-l) and $C_{k,p\ o}(t)$ is the coefficient of Z↑t in the expansion $(1-Z^P/1-Z)^K$

and μ(K,P) is chosen to satisfy the normalizing condition (37).

B. Extension to the non-stationary cage?

Zurbenko (1991) has extended some of the preceding considerations to the non-stationary case. The non-stationary process X(t) is imagined as (dependent on two parameters) X(t,u) and is assumed covariance stationary with respect to the discrete parameter t and dependent on the continuous real parameter u, which is supposed to capture slow changes in the spectrum. The process X(t,u) is assumed to satisfy a number of conditions.

(i) Uniformly bounded absolute moments exist upto (and inclusive of) the 4th order i.e.

$$E|X(t,u)|^k \le E|X|^k \quad (k = 1,2,3,4)$$

(ii) The random variables X(t,u) are continuous in the mean aqua with respect to the parameter u.

$$E|X(t,u+\Delta u) - X(t,u)|^2 \le \rho^2|\Delta u|E|X|^2$$

where p is a constant.

(iii) The 4th cumulanta C. of $(X(t_r u)$ should be continuous w.r.t. u i.e.

$$|C_4(X(t,u+\Delta u))-C_4(X(t,u))| \le \rho|\Delta u|EX^4$$

(iv) Either the Rosenblatt mixing condition (1) or the Ibragimov mixing conditions (2) and <2') are valid.

Let the covariance function of X(t,u) be denoted by

$$C(k,u) = \mathrm{Cov}\{X(t,u), X(t+k,u)\} \tag{43}$$

under the assumed conditions (i) to (iv) C(k,u) which is independent of t (in view of stationarity in t) exists and is bounded. Further both C(k,u) and the spectral density f(λ.,u> turn out to be weakly dependent on u_r as shown by Ibragimov and Linnic (1971).

$$|C(k,u+\Delta u) - C(k,u)| \leq 4\rho\, \beta_k |\Delta u|^{1/2} EX^2 \tag{44}$$

$$|f(\lambda,u+\Delta u) - f(\lambda,u)| \leq \rho\, \beta |\Delta u|^{1/2}\ EX^2 \tag{45}$$

where *f*k and *f* are from (2) and (2). In (44) and (45) we have assumed the Ibragimov mixing condition for specificity. Analogous formulae hold for the Rosenblatt condition).

Analogously to (34), the modified periodogram for X(t,u) may be defined as^

$$W_M^Q(\lambda,u) = \sum_{t=-\infty} a_M(t-Q)X(t,u)\exp(it\lambda) \tag{46}$$

(see Zurbenko (1991)).

Let the other entities occurring in (35)-(37) be modified similarly. The expression for the spectral density estimate now becomes

$$f_N(\lambda,u) = (1/T)\sum_{k=0}^{T-1} |\ W_M^{Lk}(\lambda,u)|^2$$

where L,K,T are to satisfy (33).

The asymptotic normality of $\tilde{f}_N\ (\lambda,u)$ is proved in Zurbenko (1986).v

Its mean is $f(\lambda,u)$ (the true spectral density)and its variance is

$$(2\pi f^2(\lambda,u)/T)\int_{-\pi}^{\pi} (|\phi_M^0(x)|^4/L)\ dx \tag{48}$$

where $\phi_M^0(x) = \sum^{\infty} a_M(t)\exp(itx)$. (49)

Under conditions (i) to (iv) listed above for $X(t_r u)$

$$|\mathrm{var}\ f_N(\lambda,u+\Delta u) - \mathrm{var}\ f_N(\lambda,u)|$$

$$\leq (8\rho/N)(\Delta u)^{1/2}\beta^2 EX^2 \int_{-\pi}^{\pi} |\phi_M^0(x)|^4\ dx \quad \{1+o((\Delta u)^{1/2}) + o(1/N)\} \tag{50}$$

i.e. small deviations from stationarity will have a small influence on the variances of the spectral estimates.

From an inferential point of view, primary interest:

attaches to the matrix

$$\{|W_M^{Lk}(\lambda_i, u_k)|^2\}$$

with k=l,2...T and $\lambda.\ \varepsilon(O,\eta)$

which is composed of asymptotically non-correlated elements

distributed as $f(\lambda_i, u_k)\chi^2/2 \quad (df=2)$

Stationarity inappropriate frequency bands can now be checked by standard multivariate techniques such as ANOVA (along the lines, for example, suggested by Priestley and Subba Rao <1969) for the evolutionary spectrum.

6. Conclusions:

Several approaches to the problem of estimation of a time-varying spectrum have been reviewed in this paper. The implications of these methods can be of far-reaching significance for various applied disciplines. Prom a theoretical point of view the methods of Priestley and Zurbenko have several features in common; the idea of oscillatory processes on which Priestley bases the concept . of the evolutionary spectrum loosely corresponds to "slowly changing processes", Zurbenko[1]s reliance on mixing conditions corresponds to limitations on "remote frequency dependence". A Monte Carlo investigation of stationarity tests implied by the 2 approaches is currently in progress (by the author). A further project proposes to devise stationarity tests as well as tests for structural breaks based on some of the other approaches discussed.

FOOTNOTES

1. A measure v is said to be absolutely continuous w.r.t. a measure μ if for every measurable set A with $\mu(A) = 0$, we have $v(A) = 0$

2. Although the evolutionary spectrum defined by (17) is not invariant to the choice of the family F, the integral $\int_{-\infty} dH_t(\omega) = var(X(t))$ is independent of this choice.

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