Signaling through Public Antitrust Enforcement: A Generalization

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Abstract

This note shows that the argument of Å aljanin(2017) [Å aljanin, 2017. â€œSignaling through public antitrust enforcementâ€•, Economics Letters 169, 4 - 6] that public antitrust enforcement complements private investment is robust to allowing public investment in anti-trust enforcement to be productive. However, unlike as in the case of unproductive public investment, over investment in public antitrust enforcement does not necessarily signal that the government is pro-competition: in pooling equilibria either only the anti-competition government or both types of government over invests, whereas in the separating equilibrium only the pro-competition government over invests.

Keywords: Private and public enforcement, Signaling, Antitrust

JEL Code: H1, H4, K1, L1, L4

Signaling through Public Antitrust Enforcement: A Generalization

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April 05, 2018

Abstract: This note shows that the argument of Šaljanin(2017) [Šaljanin, 2017. "Signaling through public antitrust enforcement", *Economics Letters* 169, 4 − 6] that public antitrust enforcement complements private investment is robust to allowing public investment in anti-trust enforcement to be productive. However, unlike as in the case of unproductive public investment, over investment in public antitrust enforcement does not necessarily signal that the government is pro-competition: in pooling equilibria either only the anti-competition government or both types of government over invests, whereas in the separating equilibrium only the pro-competition government over invests.

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1. Introduction

In a recent paper Šaljanin(2017) (henceforth SS in short) provides a new rational for public antitrust enforcement. SS shows that, in the presence of asymmetric information regarding the government's type – pro-competition or anti-competition, (a) the government's motive behind over-investing in anti-trust enforcement is to signal her commitment to promote market competition and (b) public antitrust enforcement complements private investment. To demonstrate these results SS assumes that public investment in antitrust enforcement is unproductive. In a follow-up extension (Section 3.1, pp. 6) SS argues that, when public investment is productive in the sense that it enhances the probability of success of private investment projects, firms will be more willing to invest after observing higher public investment. However, it is not clear whether over investment in public antitrust enforcement will signal that the government is of pro-competition type. The reason is, if public enforcement increases the probability of success of private investment projects and the government attaches a positive value to successful private projects, the anti-competition government may have a greater incentive to invest in public anti-trust enforcement than that of the pro-competition government.

In this note we revisit the results of SS by explicitly analysing the implications of public investment in anti-trust enforcement to be productive and demonstrate the following. Under asymmetric information regarding the government's type, three equilibria – a separating equilibrium and two pooling equilibria – exist depending on parametric configurations, a la SS. The separating equilibrium outcome is similar to that in SS: only the pro-competition government over invests in antitrust enforcement compared to that under symmetric information, which credibly signals the government's true type, and private investment occurs only when there is pro-competition government. However, in contrast to SS, in pooling equilibria (a) either only the anti-competition government or both types of government over invests in public antitrust enforcement compared to that under symmetric information and (b) the private firm invests irrespective of the type of the government, though it cannot update its prior beliefs. We, therefore, can say that public investment in antitrust enforcement complements private investment regardless of whether public investment is productive or not. However, in contrast to SS, the rationale behind over investment in public antitrust enforcement is not necessarily that it is a signal of the pro-competition government's support for competition, but it can as well be an outcome of the anti-competitive government's desperation to masquerade her true identity.

2. The Model

The setup is exactly the same as in Saljanin (2017), except that we allow for investment in public antitrust enforcement to be productive. There are two players, the government and a potential private investor (henceforth 'firm'). The government can be either pro-competition type (henceforth 'type-A') or anti-competition type (henceforth 'type-B'). The government's type, which is exogenously determined, is her private information. The firm's prior belief that the government is of type A is $\alpha \in (0,1)$, which is common knowledge. There are two stages of the game involved. In stage 1, the government decides the level of public investment $e\ (\geq 0)$ in public antitrust enforcement, which is publicly observable and verifiable. Next, in stage 2, the firm decides whether to invest the fixed amount $I\ (> 0)$ in a project or not, and payoffs are realized. The firm's opportunity cost of fixed investment (I) is $(1+i)I$, where $i > 0$. The project may be successful or unsuccessful. Let $p_j(e_j) \in (0,1)$, $j = A, B$, be the success probability of the project if the government is of type j, where e_j denotes public investment made by type j government. In the case of success, the firm obtains the payoff $(1+r)I$ and the government receives benefit W regardless of her type, where $r > i$ and $W > 0$. On the other hand, in the case of failure, each player's payoff is assumed to be zero. These are common knowledge.

Assumption 1: (a) $p_A(e_A) > p_B(e_B) > 0, \forall\ e_A = e_B \geq 0$; (b) $\frac{dp_j}{de_j} > 0\ and\ \frac{d^2 p_j}{de_j^2} < 0,\ \forall e_j \geq 0,\ j = A, B$; and (c) $\frac{dp_A}{de_A} = \frac{dp_B}{de_B}$, if $e_A = e_B$.

Assumption 1(a) states that, for any given level of public investment, the project is more likely to be successful under type-A government than under type-B government, which is in the same spirit as in Saljanin (2017). However, unlike as in Saljanin (2017), we consider that the success probability of the project increases at a decreasing rate in the government's investment in public antitrust enforcement, regardless of her type (Assumption 1(b)). Assumption 1(c) states that the rate of change in the success probability, due to a change in public investment from any given level, does not depend on the type of the government.

Definition 1: Let $p_j(e_j^0)(1+r)I = (1+i)I, j = A, B$.

From Assumption 1 and Definition 1, it follows that, in the case of symmetric information, the firm invests under type j government, if $e_j \geq e_j^0$; $j = A, B$. That is, e_j^0 is the minimum public investment that the type j government needs to commit in order to attract private investment. The underlying assumption is, the firm invests unless its expected gain from investing is strictly negative.

Lemma 1: Under symmetric information, the minimum public investment required to attract private investment is less for the government of type-A than of type-B: $e_A^0 < e_B^0$.

Proof: See Appendix

Assumption 2: $e_A^0 \geq 0$

Lemma 1 and Assumption 2 together imply that $e_B^0 > 0$, i.e., in order to attract private investment the type-B government needs to make a strictly positive investment in public antitrust enforcement, while in the case of type-A government the firm may invest even when there is no public enforcement of antitrust ($e_A^0 \geq 0$). We, thus, include the possibility that, in absence of any public antitrust enforcement ($e_A = e_B = 0$), private investment occurs only if the government is of type-A, as in Saljanin (2017).

Definition 2: Let $\tilde{e}_j = argmax_{e_j}[p_j(e_j)W - e_j]$, $j = A, B$.

Lemma 2: $\tilde{e}_A = \tilde{e}_B$

Proof: See Appendix

Assumption 3: $p_j(\tilde{e}_j)W - \tilde{e}_j > 0, j = A, B$.

Assumption 4: $p_A(e_A^0)W - e_A^0 > 0$ and $p_B(e_B^0)W - e_B^0 < 0$.

Assumption 5: $e_A^0 < \tilde{e}_A$ and $\tilde{e}_B < e_B^0$

Assumption 3 states that it is incentive compatible for the type-j ($j = A, B$) government to choose her unconstrained optimal level of public investment, $\tilde{e}_j$, if that attracts private investment, compared to choosing any other level of public investment. Assumption 4 implies that, under symmetric information, only type-A government would be better off by attracting private investment, compared to investing less than the minimum required amount.

Assumption 5 implies that, if the government is of type-A (type-B), her unconstrained optimal choice of public investment satisfies (does not satisfy) the firm's incentive compatibility constraint to invest: $p_A(\tilde{e}_A)(1+r)I \geq (1+i)I$, but $p_B(\tilde{e}_B)(1+r)I < (1+i)I$. Lemma 2 and Assumptions 2 & 5 together imply the following.

$$0 \leq e_A^0 < \tilde{e}_A = \tilde{e}_B < e_B^0 \tag{1}$$

Proposition 1: Suppose that Assumptions $1 - 5$ hold true. Then, the following strategies constitute the equilibrium under symmetric information, where superscript 'S' denotes the equilibrium under symmetric information.

(i)   The type-A government chooses $e_A^S = \tilde{e}_A > e_A^0$. This means that there is more than the minimum required level of public enforcement to attract private investment.

(ii)   The type-B government chooses $e_B^S = 0$. This means that there is no public enforcement.

(iii)   The firm invests only when the government is of type-A.

Proof: Consider that information is symmetric. Suppose the government is of type-$j, j = A, B$. Then, for any given e_j, in stage 2 the firm invests, if $p_j(e_j)(1+r)I \geq (1+i)I$ holds, i.e., if $e_j \geq e_j^0$ (by Definition 1); otherwise, the firm does not invest.

Now, in stage 1, the government can choose to invest or not to invest in public antitrust enforcement. If type-j government decides to invest $e_j \in [0, e_j^0)$, firm does not invest and, thus, the government's maximum payoff is zero. On the other hand, if the government decides to invest an amount so that the firm invests, her problem can be written as $max_{e_j}(p_j(e_j)W - e_j)$, subject to the constraint $e_j \geq e_j^0$. From Definition 2 and Assumption 5, it follows that the solution of this problem is given by $\hat{e}_j = \begin{cases} \tilde{e}_j, & if\ j = A \\ e_j^0, & if\ j = B \end{cases}$. However,

$p_j(\hat{e}_j)W - \hat{e}_j \begin{cases} > 0, & if\ j = A \\ < 0, & if\ j = B \end{cases}$, by Assumptions 3 and 4. Therefore, $e_j^S = \begin{cases} \tilde{e}_j, & if\ j = A \\ 0, & if\ j = B \end{cases}$.

[QED]

Asymmetric information: Under asymmetric information, given the public enforcement in antitrust (e), the firm invests irrespective of the government's type, if e satisfies (2).

$$[\alpha\, p_A(e) + (1 - \alpha)p_B(e)](1+r)I > (1+i)I \tag{2}$$

Definition 3: Let e^P: $[\alpha\, p_A(e^P) + (1-\alpha)p_B(e^P)](1+r)I = (1+i)I$.

Lemma 3: $e_A^0 < e^P < e_B^0$

Proof: See Appendix.

That is, for any $e \geq e^P$, given the firm's prior beliefs, the firm invests.

Lemma 4: (a) For all $e \in [0, \tilde{e}_B]$, $p_B(e)W - e > 0$ and (b) there exists a unique $e = \bar{e}_B \in (\tilde{e}_B, e_B^0)$ such that $p_B(\bar{e}_B)W - \bar{e}_B = 0$.

Proof: See Appendix.

Lemma 5: $p_A(\bar{e}_B)W - \bar{e}_B > 0$

Proof: See Appendix.

Lemma 6: $e^P < (=) > \tilde{e}_B$.

Proof: See Appendix.

Lemma 7: $e^P < (=) > \bar{e}_B$.

Proof: See Appendix.

When the firm observes that the government has invested e (≥ 0) amount in public antitrust enforcement, it believes that the government is of type A with probability $\mu(e)$ and is of type B with probability $1 - \mu(e)$, $\mu(e) \in [0, 1]$.

Proposition 2: Suppose that Assumptions 1 – 5 hold true and $e^P \leq \tilde{e}_B$, implying that condition (2) is satisfied at $e = \tilde{e}_B$. Then the following beliefs and strategies constitute a pooling perfect Bayesian equilibrium.

1. The firm's beliefs are given by $\mu(e) = \begin{cases} \alpha, & if\ e = \tilde{e}_B, \\ 0, & otherwise. \end{cases}$

2. The government chooses $e^* = \tilde{e}_B$, regardless of her type. This means that the type-A government chooses her symmetric information optimal level of public enforcement, whereas type-B government masquerades her true identity by overinvesting in public enforcement compared to that under symmetric information.

3. The firm invests.

Proof: First, consider the firm's beliefs as given. Then it is optimal for type-A government to choose $\tilde{e}_B$. The reason is as follows. (a) Type-A government's optimal choice under symmetric information $\tilde{e}_A = \tilde{e}_B$ (by Proposition 1(i) and Lemma 2) and (b) the firm invests even if type-B government chooses $\tilde{e}_B$, since $e^P < \tilde{e}_B$ (by supposition). It is also optimal for type-B government to choose $\tilde{e}_B$, since (a) by choosing $\tilde{e}_B$ type-B government can masquerade her type, which induces the firm to invest ($e^P < \tilde{e}_B$, by supposition), and (b) it is incentive compatible for type-B government to masquerade her true identity by choosing $\tilde{e}_B$ compared to choosing any e that reveals her true identity (when the firm invests, type-B government's payoff is maximum at $\tilde{e}_B$ (by Definition 2) and that maximum payoff is strictly positive (by Assumption 3); whereas her payoff is non-positive if she chooses any $e \neq \tilde{e}_B$). Next, given the strategies of the firm and the government, beliefs are self-fulfilling. [QED]

Proposition 3: Suppose that Assumptions $1 - 5$ hold true and $\tilde{e}_B < e^P < \bar{e}_B$, implying that condition (1) is not satisfied at $e = \tilde{e}_B$. Then the following beliefs and strategies constitute a pooling perfect Bayesian equilibrium.

1. The firm's beliefs are given by $\mu(e) = \begin{cases} \alpha, & if\ e = e^P, \\ 0, & otherwise. \end{cases}$
2. The government chooses $e^* = e^P$, regardless of her type.
3. The firm invests.

Proof: First, consider the firm's beliefs as given. Then it is optimal for the government to choose e^P, regardless of her type, due to following reasons. For any choice of type-A government $e_A \in [0,\ \bar{e}_B)$, $p_B(\bar{e}_B)W - \bar{e}_B > 0$ (by Lemma 4, Definition 2 and Assumption 1(b)), i.e., type-B will mimic Type-A's behaviour if that attracts private investment. However, $\forall e \in [0,\ e^P)$, the firm cannot update its belief and $[\alpha\, p_A(e) + (1 - \alpha)p_B(e)](1 + r)I < (1 + i)I$ (by Assumption 1 and Definition 3) and, thus, the firm does not invest. On the other hand, if type-A government chooses $e_A \in [e^P,\ \bar{e}_B)$, type-B government mimics her behaviour, but still firm invests (by Assumption 1 and Definition 3). Type A government can also choose $e_A = \bar{e}_B$, which credibly signals her true type (assuming that type-B mimics type-A only when type-B is strictly better off by doing so) and, thus,

attracts private investment (since $\bar{e}_B > e_A^0$). However, $\forall\, e > \tilde{e}_B\,, p_A(e)W - e$ is decreasing in e (by Assumption 1, Lemma 2 and Definition 2). Therefore, it is optimal for type-A government to choose $e_A = e^P$, which makes it optimal for type-B government also to choose e^P, and the firm invests. Next, take the strategies of the firm and the government as given. Then, beliefs are self-fulfilling. [QED]

That is, when $\tilde{e}_B \leq e^P < \bar{e}_B$ holds, both type-A and type-B governments over invest in public enforcement compared to that under symmetric information. Type-A government over invests in order to enhance private investment's success probability sufficiently, so that it turns out to be incentive compatible for the firm to invest in absence of any additional information. Whereas, type-B government over invests to masquerade her true type.

Proposition 4: Suppose that Assumptions $1 - 5$ and $\bar{e}_B \leq e^P$ hold true. Then the following beliefs and strategies constitute a separating perfect Bayesian equilibrium.

1. The firm's beliefs are given by $\mu(e) = \begin{cases} 1, & if\ e \geq \bar{e}_B, \\ 0, & otherwise. \end{cases}$
2. Type-A government chooses $e^* = \bar{e}_B$, whereas type-B government chooses $e^* = 0$.
3. The firm invests if $e \geq \bar{e}_B$.

Proof: Take the beliefs of the firm as given. We have $p_B(e)W - e \leq 0, \forall\, e \geq \bar{e}_B$ (by Lemma 4, Definition 2 and Assumption 1(b)), implying that type-B government will never choose any $e \geq \bar{e}_B$. Thus, by choosing any $e \in [\bar{e}_B,\ \bar{e}_A)$, where $\bar{e}_A$ is given by $p_A(\bar{e}_A)W - \bar{e}_A = 0$ ($\bar{e}_A > \bar{e}_B$, by Assumption 1 and Lemma 4), type-A government can credibly signal her true type and it is incentive compatible for her to do so compared to choosing $e < \bar{e}_B$. Since $\tilde{e}_A < \bar{e}_B < \bar{e}_A$, $e = \bar{e}_B$ calls for the lowest deviation from type-A's symmetric information optimal choice and, thus, we can argue that type-A will choose $e^* = \bar{e}_B$ and type-B will choose $e^* = 0$ in the equilibrium. The firm invests if $e \geq \bar{e}_B$, since only type-A can choose such an e and $\bar{e}_B > e_A^0$. Now, take the strategies of the firm and the government as given. Then, beliefs are self-fulfilling. [QED]

3. Conclusion

This note generalizes the analysis of SS by explicitly modelling productive public investment in antitrust enforcement. It shows that SS's result of complementarity between public antitrust enforcement and private investment is robust. However, when public investment is

productive, over investment by the government does not necessarily signal the government's commitment for pro-competition behaviour, unlike as in the case of unproductive public investment in antitrust enforcement.

Reference:

Šaljanin, Salem (2017). "Signaling through public antitrust enforcement", *Economics Letters* $169, 4 - 6$

Appendix: Proofs of Lemmas

Proof of Lemma 1:

$p_j(e_j^0) = \frac{(1+i)I}{(1+r)I}$, $j = A, B$, by Definition 1. From Assumption 1, we have (a) $p_A(e) > p_B(e), \forall e \geq 0$ and (b) $\frac{dp_j}{de_j} > 0, \forall e_j \geq 0, j = A, B$. It follows that $e_A^0 < e_B^0$.   [QED]

Proof of Lemma 2:

We have, from Definition 2, $\left. \frac{d\, p_j(e_j)}{de_j} \right|_{e_j = \tilde{e}_j} = \frac{1}{W}, j = A, B$. It implies that $\tilde{e}_A = \tilde{e}_B$, by Assumption 1(c).   [QED]

Proof of Lemma 3:

We have $e_A^0 < e_B^0$ (by Lemma 1) and $\frac{dp_j}{de_j} > 0 \, \forall e_j \geq 0$, $j = A, B$ (by Assumption 1). Then from Definition 1 it follows that (a) $[\alpha\, p_A(e_A^0) + (1 - \alpha)p_B(e_A^0)](1 + r)I < (1 + i)I$ and (b) $[\alpha\, p_A(e_B^0) + (1 - \alpha)p_B(e_B^0)](1 + r)I > (1 + i)I$. Since $[\alpha\, p_A(e) + (1 - \alpha)p_B(e)]$ is strictly increasing in e (≥ 0), we must have $e_A^0 < e^P < e_B^0$.   [QED]

Proof of Lemma 4:

We have (i) $[p_B(e)W - e]_{e=0} = p_B(0)W > 0$, since $p_B(e) \in (0, 1) \, \forall e \geq 0$; (ii) $[p_B(e)W - e]$ is strictly concave (by Assumption 1(b)) and it is maximum at $e = \tilde{e}_B$; (iii) $\tilde{e}_B > 0$ by Lemma 2, Assumption 2 and Assumption 5. It follows that (a) $\forall\, e \in [0, \; \tilde{e}_B]$, $p_B(e)W - e > 0$ and (b) $\forall\, e > \tilde{e}_B$, $p_B(e)W - e$ is strictly decreasing in e. Further, we have $p_B(e_B^0)W -$

$e_B^0 < 0$ (by Assumption 4). Therefore, there exists a unique $e = \bar{e}_B \in (\tilde{e}_B, e_B^0)$ such that $p_B(\bar{e}_B)W - \bar{e}_B = 0$. [QED]

Proof of Lemma 5:

$p_A(\bar{e}_B)W - \bar{e}_B > p_B(\bar{e}_B)W - \bar{e}_B$, by Assumption 1(a). Further, we have $p_B(\bar{e}_B)W - \bar{e}_B = 0$, by Lemma 4. [QED]

Proof of Lemma 6:

We have $0 \leq e_A^0 < \tilde{e}_A = \tilde{e}_B < e_B^0$, by (1). It follows that $[p_A(\tilde{e}_B)](1+r)I > (1+i)I$ and $[p_B(\tilde{e}_B)](1+r)I < (1+i)I$, by Definition 1. Thus, $[\alpha\, p_A(\tilde{e}_B) + (1-\alpha)p_B(\tilde{e}_B)](1+r)I < (=) > (1+i)I$. Further, (i) by Assumption 1(b), $[\alpha\, p_A(e) + (1-\alpha)p_B(e)]$ is strictly increasing in e and (ii) $[\alpha\, p_A(e^P) + (1-\alpha)p_B(e^P)](1+r)I = (1+i)I$ (Definition 3), and $e_A^0 < e^P < e_B^0$ (Lemma 3). [QED]

Proof of Lemma 7:

Inequality (1) and Lemma 4 and together imply that $[p_A(\bar{e}_B)](1+r)I > (1+i)I$ and $[p_B(\bar{e}_B)](1+r)I < (1+i)I$. Then, from Definition 3 and Assumption 1(b), it follows that $e^P < (=) > \bar{e}_B$. [QED]