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Inflation Targeting for India?

The Implications of Limited Asset Market Participation

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NCAER Working Paper

Tara Iyer*

Abstract

This paper considers the implications of an imperfect monetary transmission mechanism for optimal monetary policy choices in an open economy. The asset market channel is restricted in this paper as some agents lack financial capacity and cannot participate in asset markets. We find that while consumer price index (CPI) inflation targeting is appropriate upon a cost-push shock when all agents can borrow and save, there exists a case for stabilizing the nominal exchange rate when financial participation is low. The analysis is applied to the Indian context where monetary policy has recently been overhauled and a new CPI ininflation targeting regime is being implemented.

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1 Introduction

"My hope is that we will get to full capital account convertibility in a short number of years."

- RBI Governor Rajan, April 10 2015

Is it optimal to target CPI inflation in India while slowing down direct market interventions to stabilize the value of the rupee? In a historic overhaul of monetary policy from the previous multi-indicator approach (Subbarao, 2013), this policy was inducted on February 20, 2015 by RBI Governor Raghuram Rajan.[1] In a few years, the RBI aims to achieve a freely floating currency. Indeed, similar strategies have worked for several industrialized countries in the past (WEO, 2005). Notably, one channel through which inflation targeting (IT) works is by changing current and expected future asset prices so that agents react by adjusting their asset holdings, thereby stimulating or contracting the economy. However, despite recent efforts by the government to open up bank accounts on a large-scale basis, most Indians (three-fourths of the 1.3 billion population) lack financial capacity and do not hold assets (RBI, 2012). When the asset market channel of the monetary transmission mechnism is thus restricted, is inflation targeting still as effective?

In a normative analysis upon a cost-push shock, this paper finds that CPI inflation targeting does lead to welfare gains when 100% of agents hold financial assets. However, when not everyone can participate in the asset markets, there is a case for greater exchange rate stability. Intuitively, this is because asset income becomes less relevant for welfare purposes when financial inclusion decreases in the model. Stabilizing inflation, which erodes the value of assets, also correspondingly becomes less relevant. Thus, optimal policy prefers to prevent a deep recession at the expense of stabilizing inflation. This is achieved by manipulating the nominal interest rate to engender a lower real appreciation and hence a lower fall in output (due to perfect risk-sharing).

Though the nominal interest rate optimally decreases (to prevent too much of a decline in output) in the full asset market participation case, it falls less as financial inclusion decreases and even optimally spikes up for very high financial exclusion. However, this is all expansionary as dynamics in an economy with non-asset holders work in a loop (Bilbiie, 2008). Asset holder consumption decreases as prices unexpectedly increase. This leads to a fall in labour demand with a lower equilibrium real wage required to clear the market. The demand of non-asset

[1] http://www.wsj.com/articles/reserve-bank-of-india-gov-rajan-inflation-on-target-1409151490

holders (who have a marginal propensity to consume of one) decreases by as much as the real wage. Output falls more, and as the fraction of non-asset holders increase, wages fall by more than output. This generates a lower decline in profits, and correspondingly a small positive income effect for asset holders. A more contractionary interest rate strengthens this loop effect, and exchange rate movements in an open economy reinforce the contraction.

This paper contributes to the debate on the literature on optimal monetary policy analysis as well as inflation targeting in India. In the former literature, no previous paper has analyzed optimal monetary policy design in an open economy model with limited asset market participation. In the original New Keynesian limited asset market participation (LAMP) paper, Gali, Lopez-Salido and Valles show that when few agents can borrow and save, a standard Taylor Rule can lead to indeterminacy in a closed economy with capital accumulation. Bilbiie (2008) builds upon the Gali et al analysis to analyze determinacy and optimal policy in a closed economy without capital accumulation. He comes up with the loop effect intuition explained above (also called the IADL or Inverted Aggregate Demand Logic), derives the welfare loss function for a closed economy, and gives a limited participation threshold beyond which active Taylor Rules become indeterminate.

Eser (2009) analyzes optimal policy and indeterminacy with LAMP in a monetary union. He finds that the optimal weight on stabilizing the output gap increases with the mean and dispersion of LAMP in a currency union. Ascari et al (2011) extend Bilbiie (2008) by incorporating sticky nominal wages. They find that LAMP is less likely to alter the trade-offs faced by the Central Bank as sticky nominal wages prevent real wages from declining too much upon an aggregate demand contraction. Ascari et al also finds that a much higher share of LAMP is required to produce the IADL. Boerma (2014) finds that the threshold value at which a standard Taylor Rule leads to indeterminacy in an open economy is higher. The paper closest to the current one is Bilbiie (2008), and we differ in two key dimensions. Firstly, we take the optimal policy analysis to the open economy to bring in the impact of trade flows, and secondly we analyze simple and implementable monetary rules that can approximate the optimal policy.

We outline the literature on inflation targeting in India, which comprises of several empirical papers and fewer theoretical ones. Batini et al (2010) theoretically study the financial accelerator mechanism in the Indian context to find that a nominal peg is least suboptimal when compared to optimal monetary policy. However, Anand et al. (2010) estimate another DSGE model with a financial accelerator for India to find that exchange rate volatility in the data is too low compared to optimal policy. Banerjee and Basu (2015), in a DSGE model with capital, find that investment-specific technology shocks are more important than total factor productivity shocks in driving output variability. In an empirical paper, Mohanty and Bhanumurthy (2014) find that inflation targeting does not imply exchange rate stability in India, and in fact exchange rate stability might anchor inflation through increased credibility and sterilization. This study differs from the previous theoretical literature by studying in detail the implications for monetary policy design of limited asset market participation.

There are some caveats to the analysis. While this paper may offer some initial reasons why it might be important to consider stabilizing the exchange rate when asset market participation is limited, there is scope for further work. For instance, it would be useful to allow for limited capital mobility. This would break the perfect international risk-sharing assumption and imply that a higher real appreciation would not necessarily lead to a corresponding contraction in asset holder demand. This would weaken the loop effect, and perhaps correspondingly weaken the case for an exchange rate peg in an economy with LAMP. Incorporating sticky wages, as shown by Ascari et al (2011), might also lead to less of an optimal policy divergence from the full asset market participation case, as real wages would not fall as much upon a supply-side shock. Real wages, might, however face greater pressure to fall in an open economy compared to Ascari et al due to the additional channel of real appreciation. It remains to be seen what these extensions might imply for the optimal policy predictions of the current paper.

The rest of the paper proceeds as follows. Section 2 constructs an open economy New Keynesian model with limited asset market participation. Optimal monetary policy for this model is analyzed in Section 3. Section 4 analyzes simple rules that come close to approximating the optimal policy. Section 5 concludes, and discusses possible policy implications and extensions to the study. A technical appendix contains the linearized equilibrium, derivations of key equations, model dynamics, and sensitivity analysis.

2 Model

This section constructs an open economy New Keynesian model (based on Gali and Monacelli, 2005) with limited asset market participation. The model is cashless, and features nominal and real rigidities to introduce a role for monetary policy. LAMP is modeled via a fraction of households not being able to borrow and save in financial markets as in Gali, Lopez-Salido and Valles (2007) and Bilbiie (2008).

2.1 Households

There exists a continuum of households indexed by $i \in [0,1]$. Fraction $1-\lambda$ of the households fully optimize each period and trade in the market for state-contingent securities. However, a fraction λ are excluded from the financial markets and cannot smooth consumption by borrowing and saving in assets. They are thus not directly affected by interest rate changes. Optimizing households are denoted with superscript ^{o} while financially excluded households are denoted with superscript ^{f}.

2.1.1 Optimizing Agents

The asset holders $i \in [\lambda, 1]$ gain utility from consumption, C_t^o, and disutility from hours worked, N_t^o. Each household chooses its optimal level of consumption, leisure, and asset holdings, via the following intertemporal maximization problem

$$\underset{\{C_c^o, N_t^o, B_{t+1}\}}{Max} U_t^o = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{o1-\sigma}}{1-\sigma} - \frac{N_t^{o1+\phi}}{1+\phi} \right\}$$

$$s.t. \quad E_t Z_{t+1} B_{t+1} + E_t Z_{t+1} \Omega_{t+1} \left(V_{t+1} + P_{t+1} D_{t+1}\right) \leq B_t + \Omega_t \left(V_t + P_t D_t\right) + W_t N_t^o - P_t C_t^o \quad (1)$$

where E_0 is the conditional expectations operator, $\beta^t \in [0,1]$ is the subjective discount factor, σ is the inverse intertemporal elasticity of substitution, ϕ is the inverse Frisch elasticity of labour supply, B_t denotes the nominal value of end of period state-contingent international assets, Ω_t represents state-contingent share holdings in firms, V_t is the market value of a share, D_t is its real dividend payoff, P_t is the consumer price index (CPI), N_t^o is labour supplied, W_t denotes the nominal wage rate, and C_t^o is a CES consumption basket of domestically produced and imported non-resource goods. The budget constraint binds as preferences are locally non-satiated.

State-contingent international assets and shares pay a return of Z_{t+1} each period. We assume lack of arbitrage in the asset markets. This implies that Z_{t+1}, the stochastic discount factor, is related to the nominal interest rate as follows

$$E_t Z_{t+1} = \frac{1}{1+i_t} \quad (2)$$

The first-order conditions on consumption and international assets can be combined to yield the intertemporal Euler Equation, which relates present consumption to future discounted consumption taking into account changes in the nominal interest rate

$$E_t Z_{t+1} = \beta E_t \left\{ \frac{C^o_{t+1}}{C^o_t} \frac{P_t}{P_{t+1}} \right\} \tag{3}$$

The first-order condition on labour gives rise to the optimal labour supply equation, which equates the real wage with the marginal rate of substitution

$$w_t = N_t^{o^{\phi}} C_t^o \tag{4}$$

2.1.2 Financially-Excluded Agents

These agents $i \in [0, \lambda]$ do not participate in asset markets and live on a subsistence level basis. They solve the following static problem each period

$$\underset{\{N_t^c\}}{Max}\, U_t^f = \frac{C_t^{f\,1-\sigma}}{1-\sigma} - \frac{N_t^{f\,1+\phi}}{1+\phi}$$

$$s.t.\ P_t C_t^f \leq W_t N_t^f \tag{5}$$

As preferences are locally non-satiated, the consumption function is given directly from the binding budget constraint. The optimal labour supply condition is

$$w_t = N_t^{f^{\phi}} C_t^f \tag{6}$$

2.1.3 Optimal Consumption Basket

The CES consumption basket held by domestic households comprises of domestically produced goods, C_{Ht}, and imported production goods, C_{Ft}, from countries $j \in [0, 1]$:

$$C_t = [(1-\alpha)^{\frac{1}{\varepsilon_H}} (C_{Ht})^{\frac{\varepsilon_H - 1}{\varepsilon_H}} + (\alpha)^{\frac{1}{\varepsilon_H}} (C_{Ft})^{\frac{\varepsilon_H - 1}{\varepsilon_H}}]^{\frac{\varepsilon_H}{\varepsilon_H - 1}}$$

where α denotes the degree of trade openness (conversely, $1-\alpha$ is the degree of home bias), ε_H is the elasticity of substitution between home and foreign goods, ε_p is the elasticity of substitution between individual varieties produced in any country j, including home, and ε_F denotes the elasticity of substitution between imported goods. The associated price index is $P_t = \left[(1-\alpha) P_{Ht}^{\varepsilon_H} + \alpha P_{Ft}^{1-\varepsilon_H}\right]^{\frac{1}{1-\varepsilon_H}}$. The CES aggregator gives the following optimal demand functions for domestic and imported goods): $C_{Ht} = (1-\alpha)(P_{Ht}/Pt)^{-\varepsilon_H} C_t$ and $C_{Ft} = (1 -$

$\alpha)(P_{Ft}/Pt)^{-\varepsilon_H}C_t$.[2]

2.2 Prices and Exchange Rates

2.2.1 Relative Prices

The model equilibrium will be defined in terms of the endogenous variable

$$S_{Ht} = \frac{p_{Ft}}{p_{Ht}}$$

This is the terms of trade, or the ratio of the relative price of imports to the relative price of domestic goods. To do this, all prices are first defined in relative terms (normalized by the CPI): $p_{Ht} = \frac{P_{Ht}}{P_t}$ and $p_{Ft} = \frac{P_{Ft}}{P_t}$. For unitary elasticity of substitution between domestic and foreign goods, and imports, replace S_{Ht} in the CPI $P_t = P_{Ht}^{1-\alpha}P_{Ft}^{\alpha}$ to obtain the relative price of domestically produced goods, $p_{Ht} = S_{Ht}^{-\alpha}$. The relative price of imports is then derived as $p_{Ft} = S_{Ht}^{1-\alpha}$.

The real exchange rate, $Q_t = \frac{\varepsilon_t P_t^*}{P_t}$, which is defined as the domestic price of a foreign basket of consumption, $\varepsilon_t P_t^*$, relative to the domestic price of a domestic basket of consumption, P_t, can also be expressed in terms of S_{Ht}. Using that $\varepsilon_t P_{Ft}^* = P_{Ft}$ (PPP holds in the import goods market) and $P_t^* = P_{Ft}^*$ (the world as a whole behaves like a closed economy), we obtain

$$Q_t = S_{Ht}^{1-\alpha}$$

2.2.2 International Risk Sharing and UIP

Since capital flows freely across borders in this model and the international asset market is complete, optimizing households share risk internationally. Thus, the consumption of optimizing agents can only increase (decrease) relative to foreign consumption if the real exchange rate depreciates (appreciates). By equating the intertemporal optimality conditions of domestic optimizers and foreign households in each country j, we obtain the following international risk-sharing (IRS) condition

$$C_t^o = \nu C_t^* Q_t \tag{7}$$

where $C_t^* = \int_0^1 C_t^j dj$ denotes world consumption and we assume symmetric initial conditions so that $\nu = 1$. Combining the IRS condition with $Z_t = E_t\{Z_{t+1}\}$ and $Z_t^j = E_t\{Z_{t+1}\varepsilon_{t+1}\}$, re-

[2] $C_{Ht} = \left[\int_0^1 C_{Hit}^{\frac{\varepsilon_p-1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}}$, $C_{Ft} = \left[\int_0^1 C_{jt}^{\frac{\varepsilon_F-1}{\varepsilon_F}} di\right]^{\frac{\varepsilon_F}{\varepsilon_F-1}}$ and $C_{jt} = \left[\int_0^1 C_{jit}^{\frac{\varepsilon_p-1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}}$

spectively the domestic and foreign country j's bond pricing equations, implies that uncovered interest rate parity (UIP) holds

$$\frac{1+i_t}{1+i_t^*} = \frac{\varepsilon_{t+1}}{\varepsilon_t} \tag{8}$$

The UIP condition captures lack of arbitrage in the international asset market. It also implies that the exchange rate overshoots in this model (for example, holding constant the foreign nominal interest rate i_t^*, an increase in the current nominal interest rate will cause capital to flow in leading to a current appreciation followed by an expected future depreciation).

2.3 Firms

Firms are monopolistic and set prices in a staggered fashion. In any given period and independent of time elapsed since last reset, fraction $(1-\theta) \in [0,1]$ of (randomly selected) firms can re-optimize prices according to the Calvo pricing scheme. Fraction $\theta \in [0,1]$ of firms cannot re-optimize and instead adjust labour demand to meet changes in economic conditions. Firms that do reset prices take into account that the probability of keeping today's price k periods ahead is given by θ^k.

With production function $Y_{it} = A_t N_{it}$, each reoptimizing firm i sets its optimal reset price P_{iHt}^* as a markup, μ, over current and expected marginal costs ($MC_t A_t = W_t/P_{Ht}$), giving rise to domestic inflation. In a symmetric equilibrium, the same price is chosen by all firms ie. $P_{iHt}^* = P_{Ht}^* \forall i$. Noting that a firm that reoptimizes in period t will choose the price P_{Ht}^* that maximizes the current value of current and future profits till period $t+k$ while this price remains effective, the optimal reset price is the solution to the following problem

$$\underset{\{P_{Ht}*\}}{Max} \sum_{k=0}^{\infty} \theta^k E_t\{Z_{t,t+k}((1-\tau)P_{Ht}^* Y_{i,t+k|t} - TC_{i,t+k|t}(Y_{i,t+k|t}))\} \tag{9}$$

$$s.t.\ \ Y_{i,t+k|t} = \left(\frac{P_{Ht}^*}{P_{H,t+k}}\right)^{-\varepsilon_p} \left(C_{Hit} + \int_0^1 C_{Hit}^j dj\right) \tag{10}$$

where $Z_{t,t+k}$ is the stochastic discount factor (as households own the firms), τ is a steady state wage subsidy, C_{Hit} and $\int_0^1 C_{Hit}^j dj$ are respective demand for good i by domestic and foreign consumers in countries j, and $Y_{i,t+k|t}$ and $TC_{i,t+k|t}$ $(Y_{i,t+k|t})$ are respectively the output and total cost in period $t+k$ for a firm that reset its price in period t. Using (10), $N_t = \int_0^1 N_{it} di$, and the price dispersion index $\Lambda_t = \int_0^1 \left(\frac{P_{Ht}^*}{P_{H,t+k}}\right)^{-\varepsilon_p} di$, the aggregate production function is given as $Y_t \Lambda_t = A_t N_t$. Maximizing (9) subject to (10) yields the optimal pricing equation

$$P^*_{Ht} = P_{Ht} \left(\frac{1 - \theta \pi_{Ht}^{\varepsilon_P - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon_P}} \tag{11}$$

The aggregate domestic price level is

$$P_{Ht} = [\theta P_{Hi,t-1}^{1-\varepsilon_p} + (1-\theta) {P^*_{Ht}}^{1-\varepsilon_p}]^{\frac{1}{1-\varepsilon_p}} \tag{12}$$

Combining (11) and (12) and log-linearizing, we obtain an expression for domestic inflation as a function of expected future domestic inflation and marginal cost gaps:

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \xi \tilde{mc}^r_t \tag{13}$$

where $\xi = \frac{(1-\theta)(1-\theta\beta)}{\theta}$. The monopolistic sector faces exogenous cost-push shocks which directly increase domestic inflation without aggregate demand pressures. These shocks might arise due to channels including sticky wages creating a time varying wage markup, or food price shocks that could create an inefficient wedge between the efficient and natural rates of output. Cost-push shocks are determined relative to their steady state value V and follow the following stationary autoregressive process

$$Ln(1+V_t) - Ln(1+V) = \rho_V Ln(1+V_{t-1}) - Ln(1+V) + \varepsilon^V_t \tag{14}$$

where $\rho_V \in (0,1)$ and $\varepsilon^V_t \sim N(0, \sigma^2_V)$. Technology shocks follow a similar process.

2.4 Monetary Policy

The Central Bank follows either optimal policy or sets the interest rate according to the following generalized rule

$$\frac{1+i_t}{1+r^e} = \left(\frac{1+\pi_{Ht-1}}{1+\pi_H} \right)^{\phi_{\pi_H}} \left(\frac{1+\pi_t}{1+\pi} \right)^{\phi_\pi} \left(\frac{\Delta\varepsilon_t}{\Delta\varepsilon} \right)^{\phi_\varepsilon} \tag{15}$$

where i_t denotes the nominal interest rate, r^e is the steady-state efficient rate of interest, π_{Ht} is domestic inflation, π_t is CPI inflation, and $\Delta\varepsilon_t$ denotes the rate of change in the nominal exchange rate. These variables are set in deviation from their steady state values. The government also gives a steady-state wage subsidy to the monopolistic sector and a subsidy to equate steady-state consumption of asset and non-asset holders. The former neutralizes the monopolistic competition distortion in steady state and the latter makes the steady state efficient and equitable in the sense that $C = C^o = C^f$ (which implies that $N = N^o = N^f$ due to identical preferences).[3]

[3] The subsidy to non-asset holders allows for algebraic tractability without sacrifycing any of the main results (Gali et al, 2007)

2.5 Market-Clearing and Accounting

The demand for each monopolistic good i is

$$Y_{it} = C_{Hit} + \int_0^1 C_{Hit}^j dj \tag{16}$$

where C_{Hit} is consumption of home goods i by domestic consumers and C_{Hit}^j denotes consumption of home good i by country j.[4] The labour market is Walrasian, with the real wage moving instantly to clear demand and supply imbalances

$$N_t = \lambda N_t^f + (1-\lambda) N_t^o$$

State-contingent international and financial assets are in zero net supply since markets are complete and the $(1-\lambda)$ fraction of agents trading in them are identical. Without loss of generality, we normalize aggregate share holdings as 1, which implies that the share holdings of each optimizer are given by

$$\Omega_{t+1} = \Omega_t = \Omega = \frac{1}{1-\lambda}$$

Aggregate consumption is a weighted average of consumption by optimizers and financially-excluded agents.

$$C_t = \lambda C_t^c + (1-\lambda) C_t^o$$

The trade balance in terms of domestic output, expressed as a fraction of steady state output Y, is zero and given by $NX_t = \frac{1}{Y}\left(Y_t - S_{Ht}^{\alpha} C_t\right)$ as in Gali and Monacelli (2005).

2.6 Equilibrium

For a particular specification of monetary policy (which can be thought of as pinning down the nominal interest rate), an imperfectly competitive equilibrium for the model is given by a sequence of prices

$$\{S_{Ht}, Z_{t,t+1}, \Pi_t, \Pi_{Ht}, MC_t, \Lambda_t, \varepsilon_t\}_{t=0}^{\infty}$$

and endogenous variables

$$\{C_t^c, C_t^o, N_t^c, N_t^o, C_{Ht}, C_{Ft}, N_t, Y_t\}_{t=0}^{\infty}$$

such that all markets clear, international-risk sharing and no-arbitrage conditions hold, house-

[4] $C_{Hit} = (1-\alpha)\left(\frac{P_{Hit}}{P_{Ht}}\right)^{-\varepsilon_p}\left(\frac{P_{Ht}}{P_t}\right)^{-\varepsilon_H} C_t$, $C_{Hit}^j = \alpha\left(\frac{P_{Hit}}{P_{Ht}}\right)^{-\varepsilon_p}\left(\frac{P_{Ht}}{\varepsilon_{jt} P_{Ft}^j}\right)^{-\varepsilon_F}\left(\frac{P_{Ft}^j}{P_t^j}\right)^{-\varepsilon_H} C_t$

holds and firms optimize, taking as given exogenous processes for shocks and foreign quantities

$$\{V_t, A_t, i_t^*, P_t^*, C_t^*\}_{t=0}^{\infty}$$

The terms of trade, S_{Ht}, is the only relative price that matters for the characterization of equilibrium. The full set of linearized equilibrium conditions can be found in Appendix 7.1.

3 Optimal Policy

This section characterizes the optimal monetary policy for the framework developed in the previous section. We choose to follow the linear-quadratic approach of Benigno and Woodford (2012), which permits an exact locally linearized approximation to the true nonlinear optimal policy problem for the case of small enough disturbances. The sufficient condition for optimality of the solution is concavity of the household utility function, which is satisfied here.

The second-order approximation of household utility yields some non-zero linear terms. If not addressed, this could lead to incorrect welfare rankings (discussed further in Woodford, 2003). We thus eliminate the linear terms in the objective function by taking a second-order approximation of the equilibrium conditions and using an appropriate wage subsidy. Welfare is then correctly evaluated upto second-order by taking a first-order approximation of the equilibrium conditions.

3.1 Setup

We first solve for the efficient allocation which is the solution to a benevolent Social Planner's problem who maximizes household utility in the limiting case where prices are fully flexible and firms operate in a competitive environment. As preferences are identical, the Planner maximizes an aggregate utility function subject to the production function, international risk-sharing condition, and resource constraint. Hereafter, we focus on the special case where $\sigma = \varepsilon_H = \varepsilon_F = 1$ (the case with $\sigma \neq 1$ is in Appendix 7.4). Note also that $C_t^{fe} = C_t^{oe} = C_t^e$ since profits are zero in the efficient equilibrium.

$$\underset{\{S_{Ht}, N_t\}}{Max}\ U_t^e = LnC_t^e - \frac{N_t^{e1+\phi}}{1+\phi}$$

$$s.t.\ \ Y_t^e = A_t N_t^e$$

$$C_t^e = C_t^* S_{Ht}^{e1-\alpha}$$

$$Y_t^e = C_t^e S_{Ht}^{e\alpha}$$

The first-order conditions for this problem yield the efficient labour allocation

$$N_t^e = (1-\alpha)^{\frac{1}{1+\phi}} \tag{17}$$

Equation (17) implies that employment is constant over time in the efficient equilibrium. It happens that the corresponding efficient rate of output, $Y_t^e = A_t(1-\alpha)^{1/(1+\phi)}$, is identical to the natural rate of output for the model (which is derived by combining the flexible price resource constraint 16, international risk-sharing 7, flexible price marginal cost condition $\frac{\varepsilon_P}{\varepsilon_P - 1} A_t = w_t S_{Ht}^{\alpha}$, and real exchange rate and terms of trade definitions). The natural equilibrium is thus efficient in this model and the property of Divine Coincidence (Blanchard and Gali, 2005) holds (output gap and domestic inflation can be stabilized simultaneously upon a demand-side shock).

Over the business cycle, the Central Bank minimizes deviations of domestic inflation and the output gap from an efficient steady state where inflation is zero and output is given by $Y^e = A(1-\alpha)^{1/(1+\phi)}$. In the absence of technology innovations, the linearized efficient rate of output will not diverge from zero. Conditional upon an appropriate wage subsidy of size $\tau = \left(1 - \alpha\frac{1-\lambda(1+\phi)}{1-\lambda}\right)\frac{S_H^{\alpha}}{A}\frac{\varepsilon_P}{\varepsilon_P - 1} + 1$ that correctly eliminates the linear term in the loss function (see Appendix 7.4 for details), the problem is

$$Min_{\pi_{Ht},\tilde{y}_t} \quad W_t = -\frac{1-\alpha}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon}{\xi}\pi_{Ht}^2 + \frac{1+\phi}{1-\lambda}\tilde{y}_t^2\right\} \tag{18}$$

$$s.t. \quad \pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa\tilde{y}_t + v_t, \kappa = \xi(1+\phi) \tag{19}$$

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\left(1-\phi\frac{\lambda}{1-\lambda}\right)}(i - E_t\pi_{Ht+1} - r_t^e) \tag{20}$$

where the micro-founded loss function in equation (18) is maximized with respect to the model's aggregate supply-side (19) and demand-side constraints (20). The welfare loss function can be viewed as a generalization of those in Gali and Monacelli (2005) and Bilbiie (2008). Compared to the former, there is a greater weight on the output gap via the $(1-\lambda)^{-1}$ term. Noting that $\lambda = 0$ in Gali and Monacelli, as $\lambda \to 1$ (more financially-constrained agents in the economy) $(1-\lambda)^{-1}$ increases and for $\lambda = 1$, there is an infinite weight on the output gap. Intuitively, domestic inflation stabilization becomes progressively less relevant for welfare purposes as $\lambda \to 1$ since fewer and fewer agents hold assets (whose value is eroded by inflation).

The key difference between ours and Bilbiie's loss function (where $\alpha = 0$) is that welfare losses decrease by factor $(1-\alpha)$ because of perfect international risk-sharing. Intuitively, the slope of the New Keynesian Phillips Curve (NKPC) is flatter by factor $(1-\alpha)$ because of the reinforcing effects of the real exchange rate. The NKPC (derived by combining and log-linearizing 7, 16, and the marginal cost condition) is augmented with an exogenous autoregressive cost-push shock v_t, which directly increases domestic inflation without pressures emanating due to changes in aggregate demand. The IS Equation (20) (derived by combining and log-linearizing 3, 5, 7, and 16) is redundant for the problem. Since the nominal interest rate does not enter in the loss function, the output gap can be used as a direct instrument of monetary policy. The implied efficient rate of interest for the model is

$$r_t^e = i_t - E_t \pi_{Ht+1} - \left(1 - \phi \frac{\lambda}{1-\lambda}\right) E_t \Delta \tilde{y}_{t+1}$$

3.2 Calibration

Before numerically analyzing optimal policy, we parametrize the framework. The key parameter of interest is λ, or the fraction of households who do not participate in asset markets. Bank accounts have been opened for close to 70% of surveyed households in India (Ministry of Finance, 2012), but three-fourths of these remain unused due to lack of household financing capacity (RBI, 2012). This implies that around 75% of Indians do not borrow and save. We calibrate $\lambda = 0.7$, which is a bit lower that this. Openness to trade (in this model - percent of imports in consumption basket) is calibrated at $\alpha = 0.4$ which is a rough estimate based on imports constituting around 30% of overall GDP (WDI, 2015). The results in this study, however, are not sensitive to this specific level of openness or financial inclusion. For instance, they also hold with $\lambda = 0.3$, which is around the fraction of households for whom the government has not yet opened bank accounts.

Other parameters are less specific to the model, and set as standard in the open economy literature. Following the benchmark values set by GM - we let the household discount factor β equal 0.99, which implies a steady state real interest rate of around four percent. We set the inverse Frisch elasticity of labour ϕ at 1 to avoid simple rule equilibrium indeterminacy issues of the type analyzed in Bilbiie (2008). However, optimal policy is not qualitatively affected by lower or higher elasticity. The fraction of monopolistic producers who can reset price θ is set at 0.75, which implies an average period of around one year between price adjustments. There is unitary elasticity of substitution between home and foreign goods, ε_H, and between imported goods, ε_F. The elasticity of substitution between differentiated monopolistic goods is set at $\varepsilon_P = 6$, which implies a steady state markup of around 20%.

3.3 Gains Under Commitment

The policymaker can set an optimal plan under commitment or discretion. While minimizing (18) subject to (19) yields the optimal targeting rule under commitment (21), an analogous static problem gives the optimal targeting rule under discretion (22).[5] A policymaker operating under discretion is free to re-optimize each period, which aggravates the tradeoff between stabilizing domestic inflation and the output gap. Figures 2 and 3 in Appendix 7.2 show the stabilization bias associated with discretion where the policymaker follows the suboptimal strategy of trying to stabilize the output gap in the medium-term, without internalizing that larger deviations of the output gap at appropriate horizons lead to greater short-term stability.

$$\pi_{Ht} = -\frac{\psi}{\kappa}\left(\tilde{y}_t - \tilde{y}_{t-1}\right) \tag{21}$$

$$\tilde{y}_t = -\frac{\kappa}{\psi}\pi_{Ht} \tag{22}$$

where $\psi = \frac{\xi}{\varepsilon}\frac{1+\phi}{1-\lambda}$ represents the relative weight on stabilizing the output gap. Commitment leads to an improved trade-off here, which can be shown by solving (21) forward so that $\pi_{Ht} = \kappa\tilde{y}_t + \kappa\sum_{t=0}^{\infty}\beta^t E_t\tilde{y}_{t+1} + v_t$. This equation implies that current domestic inflation can be decreased by lowering the current output gap but also by (credibly committing) to lower future output gaps (Gali, 2008). If the private sector expects lower future output, it will revise its inflation expectations downwards, implying that lower current inflation can be achieved by a lower reduction in the current output gap.

Moreover, in contrast to discretion, the solution under commitment delivers equilibrium determinacy regardless of the degree of asset market participation in an open economy. This can be seen by combining the optimal targeting under commitment (21) with the NKPC (19) to derive the following equation, whose roots are of opposite sign as required by determinacy, regardless of the value of λ

$$E_t\tilde{y}_{t+1} = \left[1 + \frac{1}{\beta}\left(1 + \frac{\kappa^2}{\psi}\right)\right]\tilde{y}_t - \frac{1}{\beta}\tilde{y}_{t-1} + \frac{\kappa}{\psi\beta}v_t$$

3.4 Dependence on Financial Inclusion

Due to the stabilization bias under discretion, we focus on commitment hereafter. We now examine the optimal policy implications for different degrees of asset market participation. To build intuition, the optimal policy is first compared to a simple Taylor Rule of form $i_t = 2\pi_{Ht}$.

[5] $Min_{\pi_{Ht},\tilde{y}_t} \quad W_t = -\frac{1-\alpha}{2}\left\{\frac{\varepsilon}{\xi}\pi_{Ht}^2 + \frac{1+\phi}{1-\lambda}\tilde{y}_t^2\right\}$ subject to $\pi_{Ht} = \kappa\tilde{y}_t + v_t$.

Domestic inflation instantly increases, leading to a decline in consumer demand for output due to the unexpected increase in prices. Due to the contraction in aggregate demand, the Central Bank follows an expansionary monetary policy. This manifests in different directions for the nominal interest rate, depending on the level of asset market participation.

First, consider the case where all agents hold assets. It is optimal for the Central Bank to decrease the nominal interest rate to prevent too large a fall in consumption. Consumption does fall, but by less under optimal policy compared to a simple rule (Figure 4 in Appendix 7.2). This relative expansion in consumption requires lower real appreciation. The Taylor Rule, which reacts by raising the nominal interest rate too much in react to the spike in domestic inflation, causes a suboptimal decline in output. Though domestic inflation volatility is lower compared to commitment, the Taylor Rule leads to higher welfare losses as output volatility is too high.

Now, consider the case where 70% of agents cannot borrow and save. In contrast here, the interest rate rises under under optimal policy (Figure 5 in Appendix 7.3). This, however, is still expansionary via the following loop effect. Asset holders decrease consumption upon the spike in prices, leading to a fall in output. This leads to decreased aggregate employment (despite a small increase in labour supply via the substitution effect) and real wages decrease. Non-asset holders, indexed by λ, consume wage income each period and therefore decrease demand one-to-one with the fall in wages. As λ increases, real wages fall by more leading to a greater contraction in non-asset labour demand and a corresponding fall in wages and output.[6] For a high enough fraction of asset holders in the economy, the fall is wages is great enough that profits increase leading to a positive wealth effect for asset holders.

The nominal interest rate increases to produce enough contraction in the beginning of the loop so that expansion is greater in the end. This is supported by a lower real appreciation at the end of the loop. In the beginning of the loop, however, the real exchange rate reinforces the contraction and consumption declines by more initially in an open economy compared to a closed economy (openness thus leads to a greater expansion in the end). Further, akin to the full participation case, domestic inflation is allowed to rise a bit more to prevent as deep

[6] As λ increases, aggregate labour supply effectively become more inelastic as fewer people work under log utility (as the labour supply of non-asset holders is constant). Even if non-asset holders were to work (CRRA utility case, $\sigma \neq 1$), the reaction of non-asset holders to $\eta = \frac{1-\sigma}{1+\phi}$, elasticity of hours to wage, would move the real wage in different directions. Aggregate labour supply would become more elastic (so that wages decrease by less upon a contractionary shock) but the consumption of financially constrained agents would become more elastic (it would fall by more than the real wage, leading to lower aggregate demand, leading to wages decreasing by more). These effects move the real wage in opposite directions, leading to negligible difference from the log utility case (Bilbiie, 2008).

a recession as under the Taylor Rule.

Figure 1: Optimal Policy Dependence on Asset Market Participation

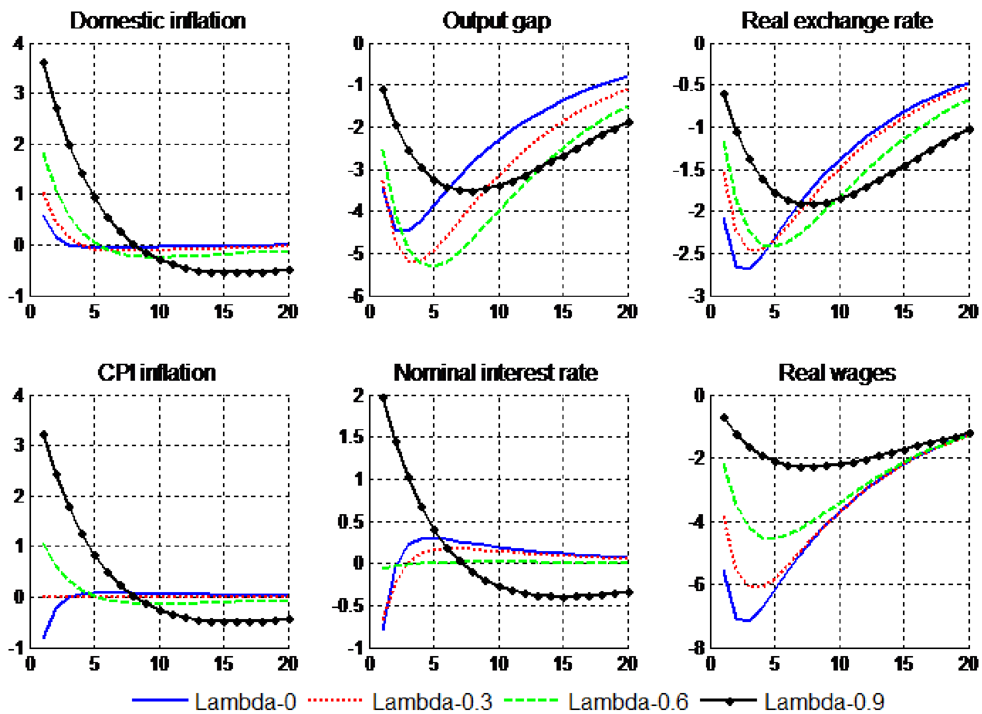


Figure 1 above analyzes how optimal policy design depends on different degrees of asset market participation. Note that an optimal expansionary stance monetary policy stance upon a cost-push shock requires a less accomodative interest rate as financial inclusion decreases. For enough asset holders, the fall in the nominal interest rate leads to a relative increase in aggregate demand but to achieve a similar relative increase in demand, the interest rate is required to increase when non-asset holders increase (to take advantage of the loop). Indeed, asset holder consumption falls by the least as λ increases due to the positive effects on profits for high enough λ.

Real appreciation is therefore required to be more contained with lower degrees of financial inclusion, which in turn leads to the lowest fall in wages via the substitution effect. Non-asset holders fully consume their wage income, implying that both asset- and non-asset holder demand falls by less as λ increases. Thus, there is a smaller recession with lower asset market participation, which is required under the optimal plan. Correspondingly, domestic inflation is allowed to increase by more since asset income becomes less relevant. CPI inflation is more volatile as financial inclusion decreases due to lower real appreciation. In the next section, we will analyze which simple rules lead to the least welfare losses compared to optimal policy for high and low levels of asset market participation.

3.5 Sensitivity

Is optimal policy robust to different calibration? The two parameters of interest are the inverse Frisch elasticity of labour supply, ϕ, and degree of openness, α. Here, we discuss optimal plan robustness with corresponding figures in Appendix 7.3.

3.5.1 Frisch Elasticity

Upon a cost-push shock, the tradeoff improves with lower labour supply elasticity (Figures 6 and 7 in Appendix 7.3).[7] Note that via standard Walrasian labour market intuition, a more inelastic labour supply leads to a greater fall in the real wage. This, ceteris paribus, implies that output decreases by the least in both full and incomplete asset market participation cases (since the cost of inputs is lower). Domestic inflation rises more (less) with elasticity in the full (incomplete) case. This is because of the competing effects on inflation of real wages and the real exchange rate.

Frisch elasticity directly impacts a fewer fraction of agents as λ increases (recall labour supply of non-asset holders is fixed - an algebraic simplification that does not affect the optimal policy analysis). The difference in initial real wage fall is therefore not as much (but real wages fall by less overall due to the expansionary loop effect). The lower real appreciation with low elasticity (which supports a less negative output gap) leads to a greater increase in inflation. When everyone holds assets, however, there is a big difference in how much the real wage declines based on the elasticity. With lower elasticity, the distinctly lower marginal costs lead to a lower rise in domestic inflation despite lower real appreciation.

3.5.2 Openness

While openness[8] does not affect the trade-off between stabilizing domestic inflation and the output gap, it causes significant divergence in consumption patterns (Figures 8 and 9 in

[7] We calibrate $\phi = 1$ in the main simulations, and in the robustness analysis, *Low Elasticity* is $\phi = 5$, *Unit Elasticity* is $\phi = 1$ and *High Elasticity* is $\phi = \frac{1}{5}$.

[8] We calibrate $\alpha = 0.4$ in the main simulations and in the robustness analysis: *Closed* is $\alpha = 0$, *Somewhat Open* is $\alpha = 0.4$ and *Very Open* is $\alpha = 0.8$.

Appendix 7.3). As α increases, the domestic consumption basket comprises of more imports. As the real exchange appreciates under optimal policy, imports are relatively cheaper and thus domestic consumption increases with α. This is supported by lower real appreciation. Non-asset holders also benefit as the economy becomes more open since lower real appreciation leads to a lower fall in real wages.

Note also that for any given level of openness, real appreciation and the fall in real wages are both lower in the limited participation case compared to the full participation economy. This is because of the expansionary loop effect when financial inclusion is not complete. Despite higher real appreciation, CPI inflation volatility is lower in both full and limited participation cases as openness increases, since the consumption basket consists of more relatively cheap import goods.

4 Inflation Targeting?

This section numerically evaluates the welfare properties of alternate simple and implementable rules compared to the benchmark optimal policy. This analysis is motivated by the fact that optimal monetary policy is difficult to implement in practice, as it often involves targeting complicated combinations of many (sometimes unobservable) endogenous variables. The rules we consider are "simple" and "implementable" in the sense that they may be considered the DSGE model equivalent of rules that policymakers follow in practice. We are specifically interested in analyzing which simple rule leads to the lowest welfare loss relative to optimal policy via the following formula

$$L_\lambda = \frac{W_\lambda^T - W_\lambda^*}{W_\lambda^*} \tag{23}$$

where W^* is the welfare loss due to optimal policy (never 0 as cost-push shocks generate a short-run trade-off), W^T is the welfare loss due to the simple rule T analyzed, and welfare losses are computed for both full ($\lambda = 0$) and incomplete ($\lambda = 0.7$) asset market participation cases. Though there is no such monotonicity in the full participation case, Table 1 shows that stabilizing the exchange rate more leads to lower welfare losses in an economy with incomplete asset market participation.[9] Moreover, strict CPI inflation targeting accompanied by a float

[9] When everyone holds assets, domestic (CPI) inflation targeting leads to an increase (decrease) in the nominal interest rate. (CPI IT requires a lower expenditure switching effect - this needs a decrease in the nominal interest rate to support a lower fall in output and hence a lower real appreciation). Including the nominal exchange rate in the targeting rule in the domestic IT case either leads to higher welfare losses in

is appropriate in the full participation case, whereas a nominal peg is least suboptimal when asset market participation is incomplete.

Table 1: Welfare Loss Relative to Optimal Policy

Simple Rule	L_0- Full Participation	$L_{0.7}$- Limited Participation
Strict IT (CPI)		
$\pi_t = 0$	**7.9**	23.6
Strict IT (domestic)		
$\pi_{Ht} = 0$	12.6	51.9
Fixed exchange rate		
$\Delta e_t = 0$	56.2	**9.6**
Flexible IT (CPI)		
$i = 1.5\pi_t$	21.8	26.1
$i = 1.5\pi_t + 2\Delta e_t$	29.4	14.8
$i = 1.5\pi_t + 4\Delta e_t$	39.6	12.6
$i = 1.5\pi_t + 6\Delta e_t$	44.2	11.7
Flexible IT (domestic)		
$i = 1.5\pi_{Ht}$	83.7	42.1
$i = 1.5\pi_{Ht} + 2\Delta e_t$	14.9	19.4
$i = 1.5\pi_{Ht} + 4\Delta e_t$	29.3	14.9
$i = 1.5\pi_{Ht} + 6\Delta e_t$	36.6	13.2

As analyzed in the previous section, the output gap should be stabilized by more as asset market participation declines. This requires lower real appreciation compared to the full participation case. A simple rule that can approximate the optimal policy for high λ is therefore the nominal peg, which leads to the most muted real appreciation. The nominal peg is, however, highly inappropriate when everyone can save and borrow. It leads to high

general, except for low weight on the exchange rate - here, the nominal interest rate rises by a bit less so that the output gap doesn't decrease as much. For higher weights on the exchange rate, output continue to fall by less but domestic inflation increases too much due to lower real appreciation.

and costly domestic inflation that erodes asset values. Domestic inflation targeting is also not appropriate since this requires too much of an increase in the relative price of domestic goods, which can only be achieved by too steep of a real appreciation. This high real appreciation leads to a suboptimal plummet of output.

CPI inflation targeting, in contrast, does not require an expenditure-switching effect of the same magnitude as domestic inflation targeting as it stabilizes both domestic and import prices. Thus, real appreciation is lower. Although this implies implies higher domestic inflation volatility, the recession is much smaller, engendering a lower trade-off in the full participation case. The case for CPI IT decreases, however, with the degree of financial exclusion.

5 Conclusion

This study analyzed the design of optimal monetary policy in an open economy DSGE model where asset market participation is limited. We find that while CPI inflation targeting, along with a free float, is least suboptimal when all agents can borrow and save, it is desirable to place greater weight on stabilizing the nominal exchange rate as asset market participation declines. These results may have some relevance for the RBI in the transition to a full-fledged IT regime with limited exchange rate interventions. Given that 75% of households in India do not participate in financial markets despite recent government efforts to open up bank accounts, it may be useful to consider delaying the transition to a fully free float. If needed to transition quickly to float, this paper implies that lower societal welfare losses would be attained if asset market participation were to increase.

The analysis in this paper comes with caveats. To focus specifically on the implications of a restricted asset market channel on monetary policy choices, we abstract from other potentially relevant empirical features. In future work, it would be useful to model limited capital mobility (which would break perfect risk-sharing and thus modify some of the results in this paper) and nominal wage rigidities (which would weaken some of the loop effects discussed) to understand their interactions with limited asset market participation and corresponding implications for optimal policy. An understanding of the dependence of optimal monetary policy in this paper on government behaviour would be further useful, especially in an era when the Finance Ministry of India is considering fiscal consolidation.

6 References

Anand, R., Peiris, S. and Saxegaard, M. (2010). An Estimated Model with Macrofinancial Linkages for India. International Monetary Fund, IMF Working Paper 10/21.

Ascari, G., Colciago, A. and Rossi, L (2010). Limited Asset Market Participation: Does it Really Matter for Monetary Policy? University of Pavia, Department of Economics and Quantitative Methods, Quaderni di Dipartimento 124.

Banerjee, S. and Basu, P. (2015). A Dynamic Stochastic General Equilibrium Model for India. National Council of Applied Economic Research, NCAER Working Paper 109.

Batini. N., Gabriel, V., Levine, P. and Pearlman, J. (2010). A Floating versus Managed Exchange Rate Regime in a DSGE Model of India. National Institute of Public Finance and Policy, NIPFP Working Paper 10/70.

Benigno, P. and Woodford, M. (2012). Linear-Quadratic Approximation of Optimal Policy Problems. Journal of Economic Theory, vol. 147(1), pp. 1-42.

Bilbiie, F. O. (2008). Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic. Journal of Economic Theory, 140(1), pp. 162-196.

Blanchard, O. and Gali, J. (2005). Real Wage Rigidities and the New Keynesian Model. NBER Working Papers 11806.

Boerma, J. (2014). Openness and the (Inverted) Aggregate Demand Logic. Netherlands Central Bank, Research Department, DNB Working Paper 436.

Gali, J. and Monacelli, T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. The Review of Economic Studies 72(3), pp. 202-734.

Gali, J., Lopez-Salido, J.D. and Valles, J. (2007). Understanding the Effects of Government Spending on Consumption. Journal of the European Economic Association, 5(1), pp. 227-270.

Gali, J. (2008) Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. Princeton University Press.

Eser, F (2009). Monetary Policy in a Currency Union with Heterogeneous Limited Asset Markets Participation. University of Oxford, Department of Economics, Economics Series Working Paper 464.

Ministry of Finance (2012). An Overview of Financial Inclusion. Department of Financial Services, Ministry of Finance, Government of India.

Mohanty, B. and Bhanumurthy, N.R. (2014). Exchange Rate Regimes and Inflation: Evidence from India. International Economic Journal, vol. 28(2), pp. 311-332.

WEO (2005). Does Inflation Targeting Work in Emerging Markets? World Economic Outlook, Washington DC: International Monetary Fund.

Rajan, R. (2013). Statement on taking office on September 4, 2013. Reserve Bank of India.

RBI (2012). Reserve Bank of India Annual Report, Reserve Bank of India.

Subbarao, D. (2013). Central Banking in Emerging Economies – Emerging Challenges. Speech Delivered at European Economics and Financial Centre, London, July 17, 2013.

Woodford, M. (2003), Interest and Prices, Princeton, NJ: Princeton University Press.

World Bank (2012). Global Financial Development Report: Financial Inclusion. Washington DC: World Bank.

7 Appendix

7.1 Equilibrium

The model is solved by taking first-order Taylor Approximations of the (stationarized) non-linear equilibrium equations around a zero inflation steady state with relative prices normalized to 1. This yields the following imperfectly competitive linearized equilibrium (where $\hat{p}_{it}$ denotes the linearized version of the relative price $p_{it} = \frac{P_{it}}{P_t}$). The only relative price needed to pin down the equilibrium is s_{Ht}, which is the linearized relative price of imported to domestically produced goods.

Households

$$c_t^o = c_{t+1}^o - (i_t - \pi_{t+1})$$

$$c_t^o = c_t^* + (1-\alpha)s_{Ht}$$

$$w_t = c_t^o + \phi n_t^o$$

$$c_t^f = \omega_t$$

$$n_t^f = 0$$

$$c_{Ht} = -\hat{p}_{Ht} + c_t$$

$$c_{Ft} = -\hat{p}_{Ft} + c_t$$

Firms

$$\pi_{Ht} = \beta\pi_{H,t+1} + \xi mc_t + v_t$$

$$mc_t = w_t - \hat{p_{Ht}} - a_t$$

$$y_t = a_t + n_t$$

Prices

$$0 = (1-\alpha)\hat{p}_{Ht} + \alpha\hat{p}_{Ft}$$

$$\pi_{Ht} - \pi_t = \hat{p}_{Ht} - \hat{p}_{Ht-1}$$

$$\hat{p}_{Ht} = -\alpha s_{Ht}$$

$$\hat{p}_{Ft} = (1-\alpha)s_{Ht}$$

Market-clearing and accounting

$$y_t = c_t + \alpha s_{Ht}$$

$$c_t = (1-\lambda)c_t^o + \lambda c_t^f$$

$$n_t = (1-\lambda)n_t^o$$

$$nx_t = y_t - \alpha s_{Ht} - c_t$$

Monetary policy

$$i_t = \phi_\pi \pi_t + \phi_{\pi_H}\pi_{Ht} + \phi_e \Delta e_t$$

Exogenous processes

$$v_t = \rho_v v_{t-1} + \varepsilon_{vt}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{at}$$

7.2 Dynamics

Figure 2: Commitment Versus Discretion (Full Participation)

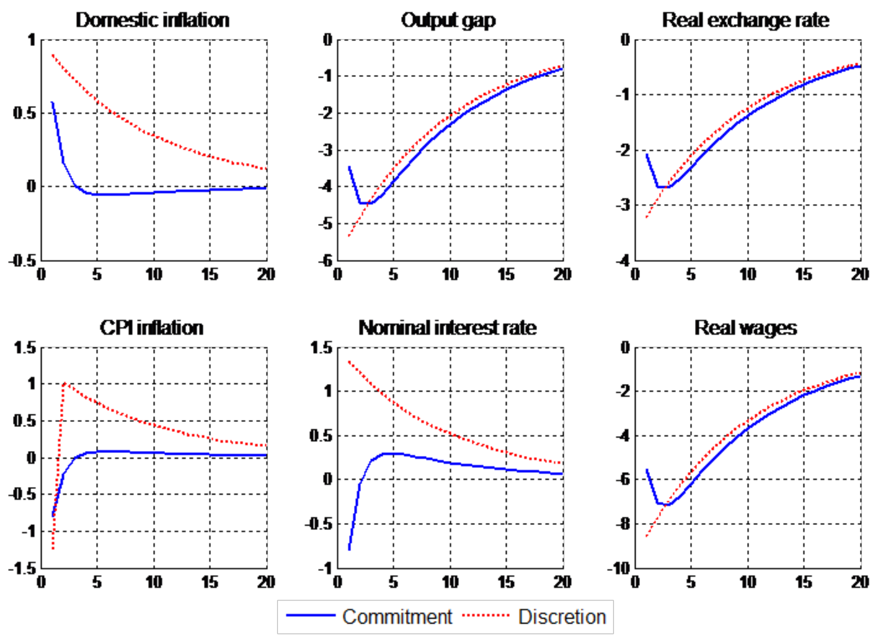


Figure 3: Commitment Versus Discretion (Limited Participation)

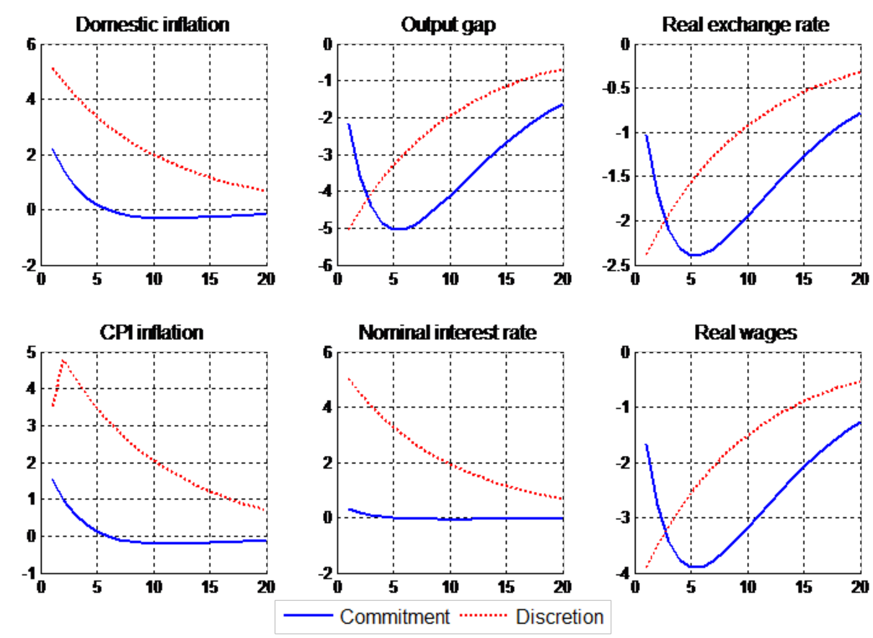


Figure 4: Commitment Versus Taylor Rule (Full Participation)

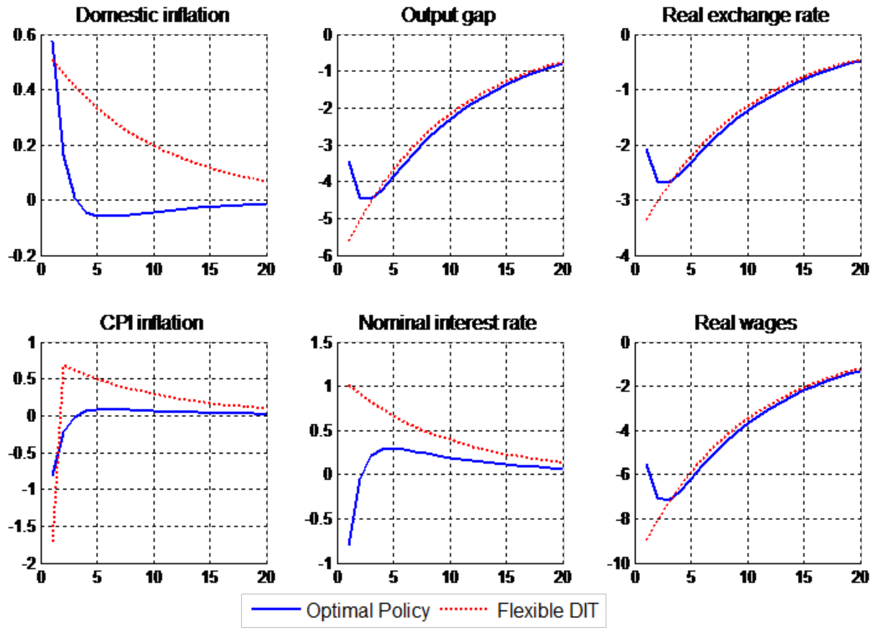
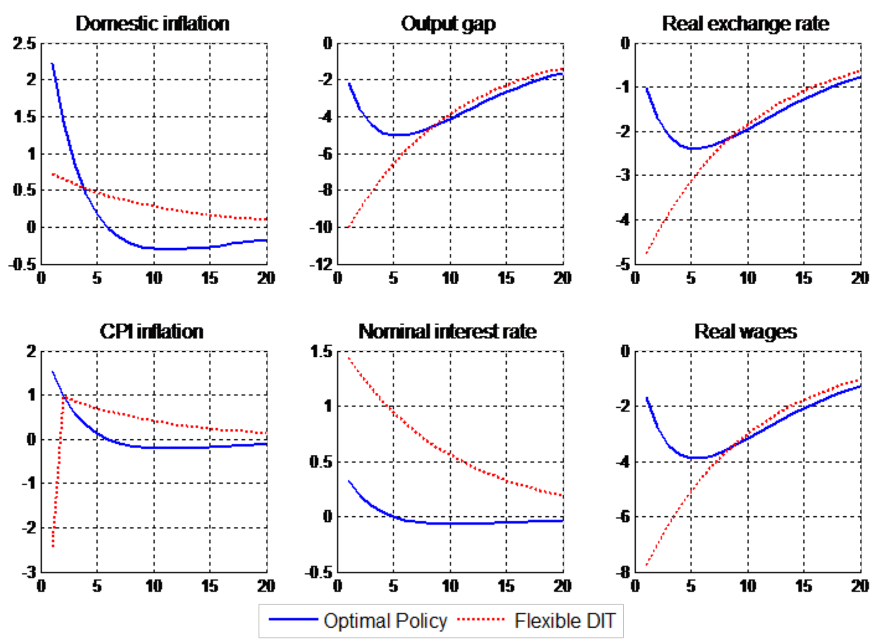


Figure 5: Commitment Versus Taylor Rule (Limited Participation)



7.3 Sensitivity

Figure 6: Frisch Elasticity (Full Participation)

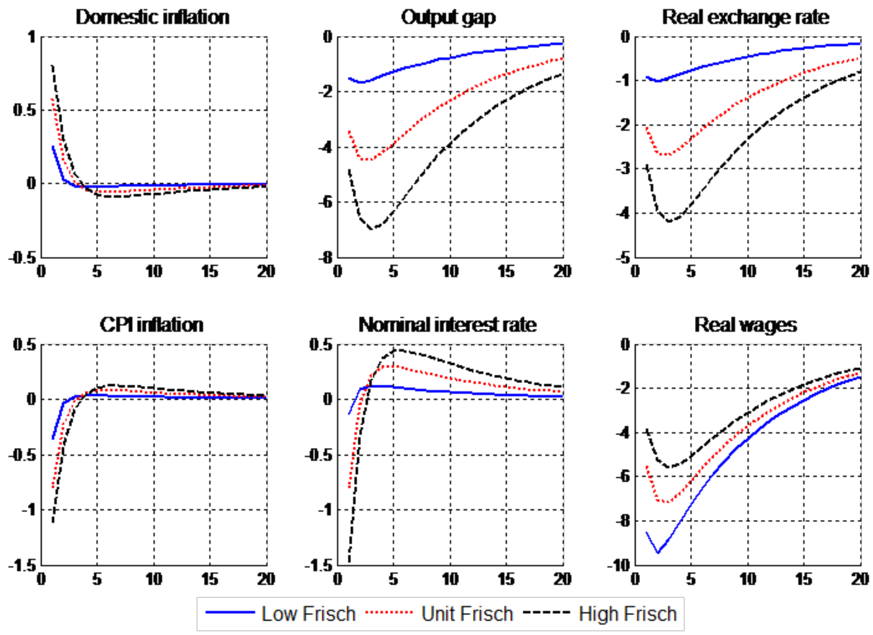


Figure 7: Frisch Elasticity (Limited Participation)

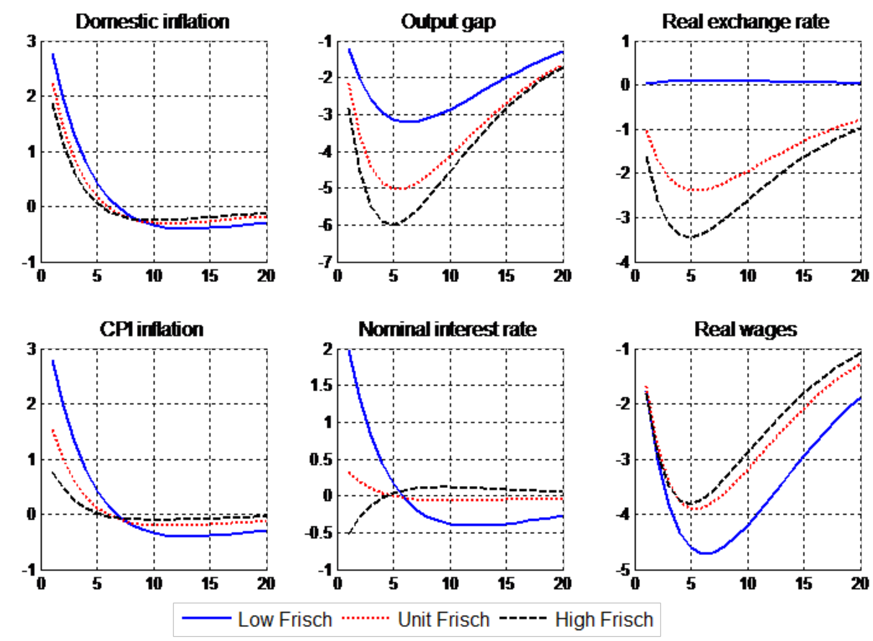


Figure 8: Openness (Full Participation)

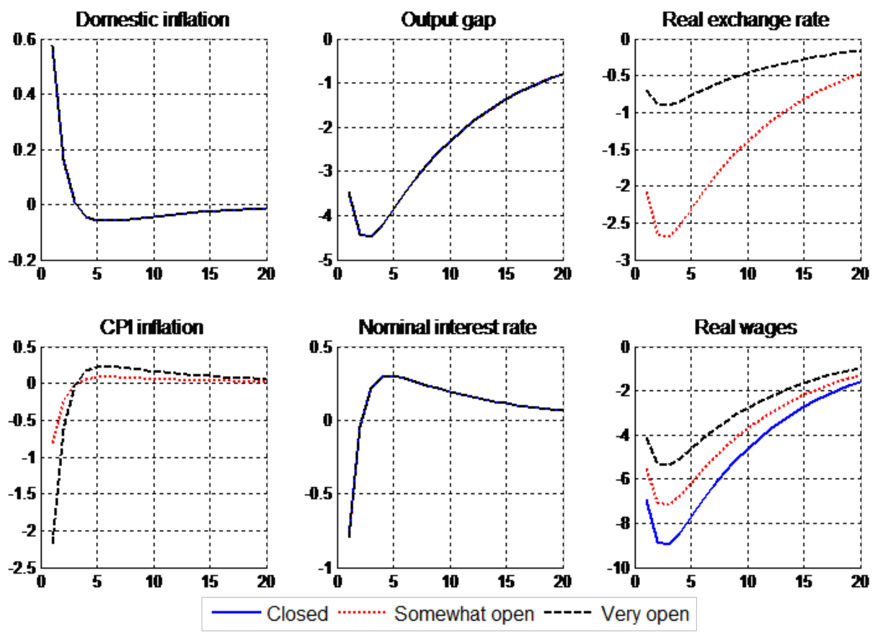
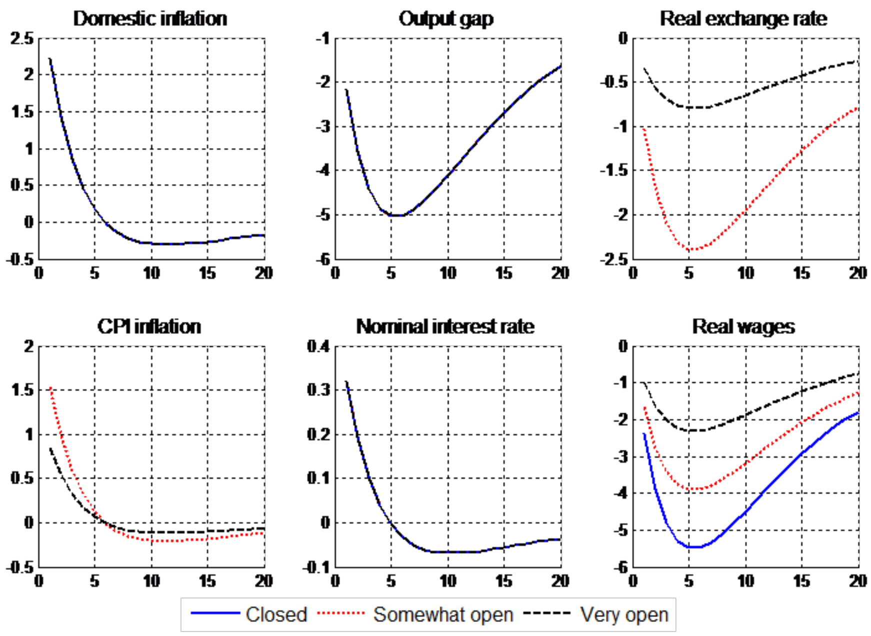


Figure 9: Openness (Limited Participation)



7.4 Loss Function

To characterize optimal monetary policy in the constrained efficient case, we follow the approach of Benigno and Woodford (2012), which eliminates linear terms in the loss function by using second-order approximations of the equilibrium conditions. The aggregate utility function is as follows with $U^i = U(C_t^i, N_t^i) = LnC_t^i - N_t^{i1+\phi}/(1+\phi)$, $i = \{o, f\}$ and weights on each household's utility determined by λ

$$U_t = \lambda U_t^f + (1-\lambda)U_t^o$$

Noting that $t.i.p.$ denotes terms independent of policy, $o(3)$ contains all terms of order higher than two, and $x_t = \tilde{x}_t + x_t^e$, it is straightforward to derive the following second-order approximation

$$\frac{U_t - U}{U_C C} = \tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t + \frac{U_N}{U_C}\frac{N}{C}\frac{(1+\phi)}{(1-\lambda)}\tilde{n}_t n_t^e + \frac{U_N}{U_C}\frac{N}{C}\frac{(1+\phi)}{2(1-\lambda)}\tilde{n}_t^2 + t.i.p. + o(3)$$

To eliminate the linear term $\frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t$, we can use the (exactly log-linear) production function $\tilde{n}_t = \tilde{y}_t + \Delta_t$ along with the result in Woodford (2003) that $\Delta_t = \frac{\varepsilon}{2\xi}\pi_{Ht}^2$. Note also that $\tilde{n}_t^2 = \tilde{y}_t^2$ and that $n_t^e = 0$ since the natural equilibrium is efficient

$$\frac{U_t - U}{U_C C} = \tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{y}_t + \frac{U_N}{U_C}\frac{N}{C}\left(\frac{(1+\phi)}{2(1-\lambda)}\tilde{y}_t^2 + \frac{\varepsilon}{2\xi}\pi_{Ht}^2\right) t.i.p. + o(3)$$

Using the (exactly log-linear) resource constraint: $Y_t = C_t S_{Ht}^{\alpha}$: $\tilde{y}_t = \tilde{c}_t + \alpha\tilde{s}_{Ht} + t.i.p + o(3)$

$$\frac{U_t - U}{U_C C} = \tilde{y}_t - \alpha\tilde{s}_{Ht} + \frac{U_N}{U_C}\frac{N}{C}\tilde{y}_t + \frac{U_N}{U_C}\frac{N}{C}\left(\frac{(1+\phi)}{2(1-\lambda)}\tilde{y}_t^2 + \frac{\varepsilon}{2\xi}\pi_{Ht}^2\right) t.i.p. + o(3)$$

Use $\tilde{y}_t = \tilde{c}_t + \alpha\tilde{s}_{Ht} + t.i.p + o(3)$, along with the (exactly log-linear) international risk sharing condition $C_t^o = C_t^* S_{Ht}^{1-\alpha}$: $c_t^o = c_t^* + (1-\alpha)s_{Ht} + t.i.p + o(3)$, (exactly log-linear) non-asset holder consumption function $C_t^c = w_t$: $c_t^c = \omega_t + t.i.p + o(3)$ and the non-linear aggregate consumption definition: $C_t = \lambda C_t^f + (1-\lambda)C_t^o$: $\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2 + \tilde{c}_t c_t^e = \lambda\tilde{c}_t{}^f + \frac{1}{2}\tilde{c}_t^{f2} + \lambda\tilde{c}_t{}^f c_t^{fe} + (1-\lambda)\tilde{c}_t{}^o + \frac{(1-\lambda)}{2}\tilde{c}_t^2 + (1-\lambda)\tilde{c}_t{}^o c_t^{oe} + t.i.p + o(3)$ to derive that $s_{Ht} = \tilde{y}_t\left(\frac{1-\lambda(1+\phi)}{1-\lambda}\right) + t.i.p + o(3)$. This yields

$$\frac{U_t - U}{U_C C} = \left(1 - \alpha\frac{1-\lambda(1+\phi)}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C}\right)\tilde{y}_t + \frac{U_N}{U_C}\frac{N}{C}\left(\frac{(1+\phi)}{2(1-\lambda)}\tilde{y}_t^2 + \frac{\varepsilon}{2\xi}\pi_{Ht}^2\right) t.i.p. + o(3)$$

A subsidy of size $(1-\tau) = \left(1 - \alpha\frac{1-\lambda(1+\phi)}{1-\lambda}\right)\frac{S_H^{\alpha}}{A}\frac{\varepsilon_P}{\varepsilon_P - 1}$ is given to make the natural equilibrium efficient and eliminate the linear term. Summing up over infinite time periods and discounting via β, the welfare loss function is

$$W=-\frac{1-\alpha}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon}{\xi}\pi_{Ht}^2+\left(\frac{1+\phi}{1-\lambda}\right)\tilde{y}_t^2\right\}$$

For CRRA utility, the optimal subsidy is of size $\tau=\left(1-\alpha\frac{1-\lambda(1+\phi)}{1-\lambda}\right)\mu\frac{S_H^\alpha}{A}\frac{\varepsilon}{\varepsilon-1}+1$ and $\mu=\frac{\sigma}{1+\alpha(w-1)}$, $\varsigma=\frac{\alpha w}{\sigma}$, $w=\sigma\varepsilon_F+(1-\alpha)(\sigma\varepsilon_H-1)$, $\eta=\frac{1-\sigma}{1+\phi}$. Following similar steps as above, we derive the following loss function

$$W=-\frac{1}{2}\frac{1-\alpha}{1-\alpha+\varsigma}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon}{\xi}\pi_{Ht}^2+\left(\frac{\sigma+\phi}{1-\lambda}\right)\Phi_y\tilde{y}_t^2\right\}$$

Here, $\Phi_y=\left(\frac{(1-\sigma)(1-\lambda)(1-\alpha+\varsigma)}{(1-\alpha)(\sigma+\phi)}\left(\lambda\Phi_{cf}+(1-\lambda)\Phi_{co}\right)-\frac{(1-\lambda)(1+\phi)}{(\sigma+\phi)}\left(\lambda\Phi_{nf}+(1-\lambda)\Phi_{no}\right)\right)$

$\Phi_{cf}=(1+\eta)\left(\phi+(1-\sigma)(1-\alpha)\mu\right)^2$, $\Phi_{nf}=\eta\left(\phi+(1-\sigma)(1-\alpha)\mu\right)^2$

$\Phi_{co}=\left(\frac{1}{1-\lambda}\right)^2\left(\mu^2(1-\alpha)^2+2\lambda\mu(1-\alpha)(1-\eta)\left(\phi+(1-\sigma)(1-\alpha)\mu\right)+(1+\eta)\left(\phi+(1-\sigma)(1-\alpha)\mu\right)^2\right)$

$\Phi_{no}=\left(\frac{1}{1-\lambda}\right)^2\left(1+2\lambda\eta\left(\phi+(1-\sigma)(1-\alpha)\mu\right)+\eta\left(\phi+(1-\sigma)(1-\alpha)\mu\right)^2\right)$

For $\sigma=\varepsilon_H=\varepsilon_F=1\rightarrow\frac{(1-\sigma)(1-\lambda)(1-\alpha+\varsigma)}{(1-\alpha)(\sigma+\phi)}\left(\lambda\Phi_{cf}+(1-\lambda)\Phi_{co}\right)=0$, $\Phi_{nf}=0$, and $\Phi_{no}=\frac{(1-\lambda)(1+\phi)}{(1+\phi)}(1-\lambda)\left(\frac{1}{1-\lambda}\right)^2=1$. So $\Phi_y=1$ (and also $\varsigma=\alpha$ and $\mu=1$) so that the loss function with CRRA utility exactly boils down to the log utility case.