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A DYNAMIC ECONOMIC MODEL OF SOIL CONSERVATION INVOLVING GENETICALLY MODIFIED CROP

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Abstract

This paper attempts to model the positive role of cultivation of Genetically Modified (GM) crop with its soil-anchoring root-characteristic and use of conservation-tillage technology, in saving organic matter contents in the topsoil and reducing soil erosion. In a dynamic optimization framework the farmer produces an optimal combination of a GM and a Non-GM variety of the same crop at the steady state, though the steady state is approached most rapidly by producing a single crop. The improvement in the capacity to anchor the soil and an increase in organic matter content in top-soil will raise the long run soil stock under certain conditions. However, the policies to increase R&D investment in genetic modification and imposition of an input subsidy on GM sector will lead to an increment in area under GM cultivation though their effect on long run soil stock is uncertain.

Keywords: *Dynamic Optimization, Genetically Modified crops, soil erosion, soil conservation, steady state.*

JEL Codes: *C61, C62, Q2, Q16, Q28*

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INTRODUCTION

The problem of soil erosion in the cultivated land is of great concern for years. Inappropriate agricultural production technology and indiscriminate use of chemical pesticides can lead to loss of topsoil through erosion, loss of organic matter through oxidation, soil compaction, loss of nutrients, accumulation of salt and trace elements and soil toxicity. It also increases runoff of fertilizers and pesticides to surfaces and ground water. Thus soil degradation has both direct and indirect negative effects on soil productivity and the environment (through water pollution) respectively. Therefore, soil conservation has to be an essential part of crop production to ensure agricultural sustainability, which means the attainment and continued satisfaction of human needs for both current and future generation. There have been a number of traditional methods which are part of soil conservation management system, which help to maintain and improve soil resource. Use of rotational cropping pattern, crop residue management and conservation buffer and structures are the most eminent examples of traditional conservation practices (Magleby, 2002).

Genetically modified herbicide tolerant crops, a product of modern plant biotechnology, may fruitfully be utilized in the crop residue management practices especially in low-till soil conservation farming system. The tillage system, that leaves substantial amount of crop residue on the soil surface, reduce erosion through wind and rainfall, increase water filtration, moisture retention and raise the level of organic matter in the top soil (Dick and Daniel, 1987; Edwards, 1995). By introducing herbicide tolerant GM crops in conservation tillage program (i.e., .no-till or low-till) farmers can apply herbicides, if needed, to control weeds without affecting the main GM crop and get economic benefit. For effective utilization of conserved soil stocks, impetus has been given on plant root characters (Skaggs and Shouse, 2008). Therefore, GM crops having longer and voluminous roots will be beneficial for binding and absorbing nutrient particles with the soil. GM crops having this specific root structure would be needed in no-till farming in the situation if the conserved moisture and nutrients are in variable depths of the soil. The present paper introduces a new approach in the traditional soil management system of conservation tillage through the involvement of herbicide tolerant GM crops with specific root characters, using a dynamic economic modeling, as soil management is a dynamic process.

The pioneering contribution in the area of dynamic economic modeling of soil conservation is by Oscar Burt (1981) who applied dynamic optimization technique in the economics of soil conservation. Taylor et al. (1986), however, criticized Burt's work as he did not consider the conservation tillage or other tructural activities as decision variable in erosion control. McConnell (1983) formulated a theoretical model of optimal control to derive the optimal inter-temporal path of soil use and suggested that in presence of an efficient capital market and equal private and social discount rate, the private inter-temporal path of soil use will converge to that of the society. McConnell's work has been criticized on the ground that in presence of market imperfection or non-existence of market in developing countries or even in presence of perfect market the private and social discount rates may not be equal (Kiker and Lynne, 1986). Walker (1982) formulated a damage function to evaluate the profitability of conservation tillage over the erosive activities of a single crop growing farmer and later incorporated the long-run cost of erosion in that function and also its empirical application to evaluate conservation tillage (Walker and Young, 1986). Barrett, 1991; La France, 1992; Clarke, 1992 and Hu et al., 1997 extended McConnell's model to examine whether the change in output or input price affect the farmer's optimizing behavior for soil conservation. With the proposition that the rate of soil loss can be curbed by having a diversified production system (without mentioning criteria of choosing a crop in terms of any specific soil-conserving feature), Goetz (1997) have derived "the optimal private and social inter temporal path of soil use" and the determined the optimum crop mix. A rather recent work of Lankosky et al. (2006) examines both theoretically and empirically the private profitability and social desirability of conventional tillage and no-till farming by taking into account crop yield, production cost and nutrients and herbicide run-off damages. The prominent empirical works showed (i) the implications of the of agricultural policies on land degradation and soil quality improvement from input –output data via a dynamic production model (Kim et al., 2000), (ii) the role of sustainable agricultural practices with respect to soil conservation (Castano et al , 2005), (iii) the need to incorporate the role of output diversification, land tenure, and human capital formation as effective instruments in increasing farm income, determined simultaneously by farmer's decision to adopt soil conservation, while framing the investments in natural resource management projects by governments and multilateral development agencies (Bravo-Ureta et al, 2006), (iv) the importance of the policies for efficient land-use and management through technology development (Pannell ,2009).

In the existing literature on the economics of soil conservation, the use of GM crop cultivation as part of output diversification has not been taken under consideration. In the present paper it is assumed that farmers cultivate both GM and non–GM variety of different or of the same crop in specific situations. Such an assumption of co-existence of GM and Non-GM crop is not an unrealistic one given the fact that the overall real world experience shows that GM crops have successfully coexisted with conventional and organic crops (Brooks, 2004)[1]. This paper may be considered as the first attempt to model the impact of a particular

[1] Coexistence as an issue relates to the principle that the farmers should be able to cultivate freely the agricultural crops they choose, be it GM crops, conventional crops or organic crops using the production system they prefer (Commission of the European Communities, 2003).There are evidences from EU of the co-existence between GM and non-GM in a viable way. In Spain, 20-25000 ha of GM maize is cultivated by farmers in areas where conventional and organic maize are also produced (Schiemann, 2003). In North America the states like Iowa and Minnesota with the greatest concentration of organic soybeans and maize are also the states with above average penetration of GM crop (Brooks, 2004).

feature such as the soil-anchoring longer and voluminous roots of a GM crop and use of conservation tillage technology in improving the long run soil stock. . Applying Goetz's model of crop diversification we determine the optimum cropping pattern of a farmer who cultivates a mix of GM and non-GM varieties of the same or different crops. The paper has also addressed a few important policy issues such as the role of the R&D investment in genetic modification and imposition of input subsidies. Though any specific area of operation is not mentioned here, this theoretical model is applicable to rain-fed and irrigated soils for both conservation and efficient use of conserved stock.

The paper is structured as follows. Section 2 and 3 present the economic model and the producer's optimization exercise whereas section 4 and 5 contain the stability and policy analyses respectively. The conclusions are drawn in section 6.

THE ECONOMIC MODEL

Let crop 1 be the GM crop and crop 2 be the Non-GM variety of the same/different crop. The per hectare production function of a crop is given by

$$q_i = f^i(s, z_i, \phi^i(M_i, R_i)) \qquad (1)$$

where q_i = output of the *i* th crop, s = overall soil depth , z_i = index of inputs, $\phi^i(M_i, R_i)$ = state of technology which is a function of M_i i.e. % of organic matter in the topsoil and R_i i.e. R& D investment in developing the *i* th crop. Burt (1981) has used depth of topsoil and the percentage of organic matter in the top 6 inches of the soil as the two state variables in his dynamic model as these two are the variables directly associated with soil erosion, even if he admits that there are many other variables contributing to plant nutrients and soil chemistry.

Here the overall soil depth is used as a state variable in the dynamic optimization problem (discussed later) and the percentage of organic matter in the topsoil has been introduced in the state of technology as it is supposed to reflect the impact of conservation tillage. Here we note that, due to the use of this low or no-till farming in GM crop production, the organic content in the land devoted to GM crop will be much higher than that of its Non-GM counterpart, which does not use any soil conserving technology. Now, following McConnell (1983) and Goetz (1997), the composite input index has been incorporated in the production function. An alternative approach can be followed, as McConnell mentioned, by dividing the composite input vector into two types: productive inputs those increase soil loss and soil conserving inputs those prevent soil depletion. As we are mainly focusing on the soil conserving technologies, any such complication is avoided.

The assumptions regarding the producion functions are as follows:

$$f_s^i > 0,\ f_{z_i}^i > 0,\ f_{ss}^i < 0,\ f_{z_i z_i}^i < 0,\ f_{\phi_i}^i > 0,\ f_{\phi_i \phi_i}^i < 0, \phi_{M_i}^i > 0, \phi_{R_i}^i > 0$$

$$f_{sz_i}^i > 0,\ f_{s\phi^i}^i > 0,\ f_{z_i\phi^i}^i > 0,$$

$$f_{ss}^i . f_{z_i z_i}^i - (f_{sz_i}^i)^2 > 0, |D| < 0 where D = \begin{vmatrix} f_{ss}^i & f_{sz_i}^i & f_{s\phi^i}^i \\ f_{z_1 s}^i & f_{z_i z_i}^i & f_{z_i\phi^i}^i \\ f_{\phi^i s}^i & f_{\phi^i z_i}^i & f_{\phi^i\phi^i}^i \end{vmatrix} \quad (2)$$

$$f^i(\bar{s}, z_i, \phi^i(M_i, R_1)) = f(s,0) = 0$$

$$f_s^i(s, z_i, \phi^i(M_i, R_i)) = 0, \forall s \geq \tilde{s}$$

where the subscripts imply the partial derivatives with respect to the variable. The production function is strictly concave in s, z_i and ϕ^i such that D is a negative definite Hessian. Here we have assumed that marginal productivity of the inputs increase with more soil and improvement in technology such that $f^i_{sz_i} > 0$ and $f^i_{z_i\phi^i} > 0$.Moreover the inputs, technology and soil are essential for production. The minimum soil depth required for agricultural production is given by $\bar{s}$, whereas beyond a certain soil depth $\tilde{s}$ production will not increase with soil depth.

The low organic content of soil reduces the infiltration and permeability as well as the stability of the soil particles such that soil can easily be destroyed byrain or wind leading to increased run-off and erosion of subsoil (Troeh et al., 1991). Hence we introduce the following per hectare *soil erosion function* which can be affected by the soil depth, inputs used as well as the root characteristics of the respective plant:

$$h^i = h^i(s, z_i, \psi^i(M_i, \theta R_{c_i})) \quad (3)$$

where $\psi^i(M_i, \theta R_{c_i}) =$ efficiency factor or state of technology that is a function of % of organic matter in the topsoil and the root characteristic (R_{c_i}) of the respective plant with θ being the % of that feature of the root that helps to prevent soil erosion. We assume the following characteristics of the erosion function:

$$h^i_s < 0, h^i_{ss} > 0, h^i_{z_i} > 0, h^i_{z_i z_i} > 0$$
$$h^i_{\psi^i} < 0, h^i_{\psi^i\psi^i} > 0, \psi^i_{M_i} > 0, \psi^i_\theta > 0$$
$$h^i_{sz_i} < 0, h^i_{z_i\psi^i} < 0, \quad (4)$$
$$h^i_{ss}.h^i_{z_i z_i} - (h^i_{sz_i})^2 > 0, |D_1| > 0 where D_1 = \begin{vmatrix} h^i_{ss} & h^i_{sz_i} & h^i_{s\psi^i} \\ h^i_{z_1 s} & h^i_{z_i z_i} & h^i_{z_i\psi^i} \\ h^i_{\varphi^i s} & h^i_{\psi^i z_i} & h^i_{\psi^i\psi^i} \end{vmatrix}$$
$$h^i(s,0) = 0, h^i_s(s, z_i) = 0, \forall s \geq \breve{s}.$$

The erosion function is strictly convex in s, z_i and ψ^i such that D_1 is a positive definite Hessian. As mentioned earlier soil depth reduces erosion such that $h_s^i < 0$. Increase in input use and tillage makes soil more erosion prone suggesting $h_{z_i}^i > 0$. The structural change of the soil due to rain aggravates the soil erosion with decrease in soil depth (Troeh et al., 1991) leading to $h_{ss}^i > 0$.With intensification of input use erosion increases, though a simultaneous addition of soil prevents that to some extent such that $h_{sz_i}^i < 0$. As far as the technology is concerned both organic matter content and the root characteristic help to prevent the soil erosion as reflected in $h_{z_i\psi^i}^i < 0$. But it can be noted here that the root characteristic of GM crop helps much more than that of the Non-GM variety to prevent soil erosion. Finally, an accumulation of soil after $\breve{s}$ does not change the magnitude of soil loss.

Here we must take note of the fact that neither the production function nor the erosion function includes the soil conserving technological aspects, such as low-till farming and crops with specific root character, in the earlier works of Clarke(1992), LaFrance(1992) and Goetz(1998), though soil conserving inputs are considered. We must also highlight the fact that the role of R&D investment in agriculture was absent in all these literatures which has captured our attention and has been included in the production function since the required genetic modification calls for a lot of research.

Since the land is divided between two segments to cultivate the two varieties of the same crop, we assume that x is the percentage of land devoted to GM variety whereas (1-x) is the devoted to the Non-GM variety. We also assume that $f^1 \neq f^2$ and $h^1 \neq h^2$.

Following Goetz we introduce the soil genesis function:

$$G = G(s), G_s < 0, G_{ss} < 0, G(\hat{s}) = 0 \quad (5)$$

According to Troeh et al (1980) genesis of soil is a decreasing function of soil depth suggesting $G_s < 0, G_{ss} < 0$. $\hat{s}$ is the level of soil depth beyond which soil does not grow any more and $\hat{s} > \tilde{s}, \breve{s}$.

The dynamics of the soil can be stated as follow

$$\dot{s} = -h^1(s, z_1, \psi^1(M_1, \theta R_{c_1}))x - h^2(s, z_2, \psi^2(M_2, \theta R_{c_2}))(1-x) + G(s) \quad (6)$$

PRODUCER'S OPTIMIZATION

The producer maximizes the present discounted value of net returns from cultivation. Thus the farmer's problem can be formulated as

max

$$\int_0^T e^{-\delta t}\left[\left\{p_1 f^1(s, z_1, \phi^1(M_1, R_1)) - p_{z_1} z_1\right\}x + \left\{p_2 f^2(s, z_2, \phi^2(M_2, R_2)) - p_{z_2} z_2\right\}(1-x)\right]dt$$

subject to

$$\dot{s} = -h^1(s, z_1, \psi^1(M_1, \theta R_{c_1}))x - h^2(s, z_2, \psi^2(M_2, \theta R_{c_2}))(1-x) + G(s)$$

$$s(0) = s_{0,} z_i \geq 0, i = 1,2, x \in [0,1] \quad (7)$$

where x, z_1, z_2 are the control variables and s is the state variable. δ >0 is the private rate of discount, $p_{z_i} > 0, i = 1,2$ is the per unit cost of input, $p_i, i = 1,2$ is the constant price of crops 1 and 2. Here, we want to assume that, there is no additional effort and cost incurred due to the joint production of GM and non-GM varieties.

The current value Hamiltonian is given by

$$H = \left[p_1 f^1(s,z_1,\phi^1(M_1,R_1)) - p_{z_1} z_1\right]x + \left[p_2 f^2(s,z_2,\phi^2(M_2,R_2)) - p_{z_2} z_2\right](1-x) + \lambda[-h^1(s,z_1,\psi^1(M_1,\theta R_{c_1}))x - h^2(s,z_2,\psi^2(M_2,\theta R_{c_2}))(1-x) + G(s)] \quad (8)$$

Taking into account the constraints on the control variable we get the Lagrangian:

$$L = H + \alpha_1(1-x) + \alpha_2 x$$

The optimal values of the control variables are associated with the costate variable $\lambda(t)$ and the Lagrange multipliers $\alpha_i(t), i = 1,2.$ The solution of the problem must satisfy the following necessary conditions:

$$L_x = \left[p_1 f^1(s,z_1,\phi^1(M_1,R_1)) - p_{z_1} z_1\right] - \left[p_2 f^2(s,z_2,\phi^2(M_2,R_2)) - p_{z_2} z_2\right] - \lambda[h^1(s,z_1,\psi^1(M_1,\theta R_{c_1})) - h^2(s,z_2,\psi^2(M_2,\theta R_{c_2}))] - \alpha_1 + \alpha_2 \quad (9)$$

$$L_{z_1} = \left[p_1 f^1_{z_1}(s,z_1,\phi^1(M_1,R_1)) - p_{z_1} - \lambda h^1_{z_1}(s,z_1,\psi^1(M_1,\theta R_{c_1}))\right]x \quad (10)$$

$$L_{z_2} = \left[p_2 f^2_{z_2}(s,z_2,\phi^2(M_2,R_2)) - p_{z_2} - \lambda h^2_{z_2}(s,z_2,\psi^2(M_2,\theta R_{c_2}))\right](1-x) \quad (11)$$

$$\dot{s} = L_\lambda = [-h^1(s,z_1,\psi^1(M_1,\theta R_{c_1}))x - h^2(s,z_2,\psi^2(M_2,\theta R_{c_2}))(1-x) + G(s)] \quad (12)$$

$$\dot{\lambda} = \delta\lambda - L_s = \delta\lambda - \left[p_1 f^1_s(s,z_1,\phi^1(M_1,R_1))\right]x - \left[p_2 f^2_s(s,z_2,\phi^2(M_2,R_2))\right](1-x) + \lambda[h^1_s(s,z_1,\psi^1(M_1,\theta R_{c_1}))x + h^2_s(s,z_2,\psi^2(M_2,\theta R_{c_2}))(1-x) - G_s(s)] \quad (13)$$

Moreover the optimal values of the control variables and the Lagrange multiplier must satisfy the Kuhn-Tucker conditions $L_{\alpha_i} \geq 0, \alpha_i \geq 0$, $\alpha_i L_{\alpha_1} = 0, \forall i = 1,2$ and $L_{z_i} \leq 0, z_i \geq 0$, $z_i L_{z_1} = 0, \forall i = 1,2$. The constraint qualification is satisfied due to the linearity of the restriction in x and z_i , i=1,2. The transversality conditions are

$$\lambda(T) \geq 0, s(T)\lambda(T) = 0 \qquad (14)$$

λ can be interpreted as the shadow price or user cost of soil depth. Therefore the necessary condition (9) for $\alpha_1 = \alpha_2 = 0$, implies that the optimal allocation of land between two varieties requires the difference between the net return and the cost of soil erosion to be same for both the varieties. The conditions (10) and (11) state that with z_i >0, the value of marginal product of input should equal the marginal cost of inputs and soil erosion.

Existence of the Optimal Control

To verify the sufficient condition for optimization we rewrite the Hamiltonian as:

$$H = \left[p_1 f^1(s, z_1, \phi^1(M_1, R_1)) - p_{z_1} z_1\right]x + \left[p_2 f^2(s, z_2, \phi^2(M_2, R_2)) - p_{z_2} z_2\right](1-x) +$$
$$\lambda[-h^1(s, z_1, \psi^1(M_1, \theta R_{c_1}))x - h^2(s, z_2, \psi^2(M_2, \theta R_{c_2}))(1-x)] + \lambda G(s)$$

Here we note that, $f^i(s, z_i, \phi^i(M_i, R_i))$ is strictly concave and $h^i(s, z_i, \psi^i(M_i, \theta R_{c_i}))$ is strictly convex in state and control variables. $G = G(s)$ is also strictly concave in the state variable. Since negative of a convex function is a concave function, H, being sum of concave functions, is strictly concave in state and control variables with $\lambda(T) \geq 0$. Thus the Mangasarian sufficiency condition (Chiang, P-214) is satisfied. This condition along with the boundary conditions guarantee the existence of unique optimal solution to this control problem and the necessary conditions in (10)-(14) characterize the optimal solution to the problem.

The Optimal Path

The linearity of H in x allows us to define a switching function, σ for the determination of the optimal trajectories of x. The function is given by:

$$H_x \equiv \sigma \equiv \left[p_1 f^1(s, z_1, \phi^1(M_1, R_1)) - p_{z_1} z_1\right] - \left[p_2 f^2(s, z_2, \phi^2(M_2, R_2)) - p_{z_2} z_2\right] - \lambda[h^1(s, z_1, \psi^1(M_1, \theta R_{c_1})) - h^2(s, z_2, \psi^2(M_2, \theta R_{c_2}))] >=< 0 \tag{15}$$

and implies that

$$x = \begin{pmatrix} 1, \sigma > 0 \\ x \in [0,1], \sigma = 0 \\ 0, \sigma < 0 \end{pmatrix} \tag{16}$$

When the net return minus cost of soil erosion for crop variety 1 exceeds that of crop variety 2 the farmer chooses the first one. The choice of variety 2 can be explained similarly. If the net returns minus costs are equal for both the crops then x is undetermined within the interval [0,1].

From the theory of optimal control it is known that the singular path ($\sigma = 0, \dot{\sigma} = 0$), if exists will be attained by the Most Rapid Approach Path (MRAP) (Hartl and Feichtinger, 1987). Accordingly we reach the first proposition:

Proposition 1: *A singular path for* $x \in [0,1]$ *is given by*

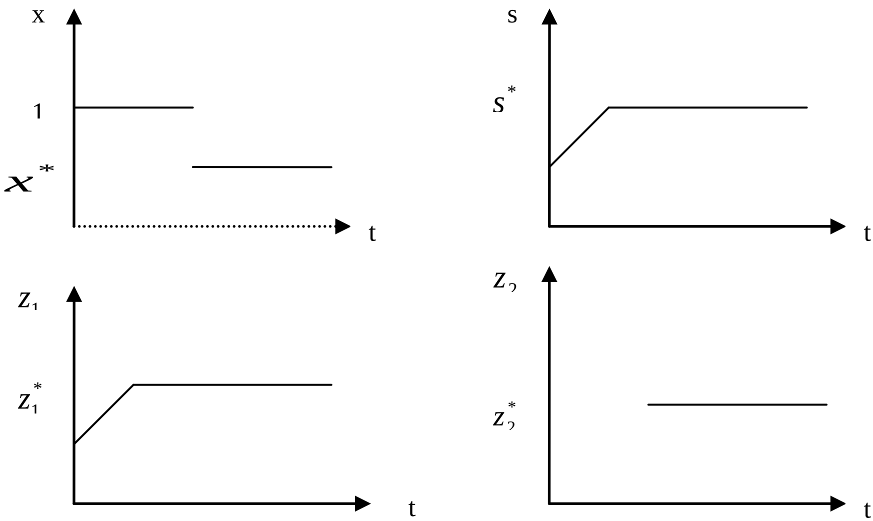
$$\left[p_1 f_s^1(s, z_1, \phi^1(M_1, R_1))\right] x + \left[p_2 f_s^2(s, z_2, \phi^2(M_2, R_2))\right](1 - x) = \lambda[\delta + h_s^1(s, z_1, \psi^1(M_1, \theta R_{c_1}))x + h_s^2(s, z_2, \psi^2(M_2, \theta R_{c_2}))(1 - x) - G_s(s)] \tag{17}$$

It is the same condition which we get in the Steady state of the system of equations (12) and (13) subject to (9)-(11). No other singular path *exists other than the steady state provided* $H_{xs}/H_{x\lambda} = -\dot{\lambda}/\dot{s}$ *holds over the entire planning horizon.*

The proof of the above proposition is given in the appendix. The economic interpretation of the condition suggests that value of the marginal increment in soil depth equals the marginal cost of soil erosion where the cost includes the private discount rate as well as the marginal erosion minus the marginal soil genesis evaluated at shadow price of the soil.

We now analyze the optimal trajectories for x, z_1, z_2 and s in Figure 1. Here we impose the restriction that $h^1 < G < h^2$ which means the GM variety saves the land from getting eroded to a greater extent compared to the Non-GM variety. For $\sigma > 0, s_0 < s^*$, it is optimal to build up the soil stock with the cultivation of GM variety until s^* i.e. the steady state value of the variable is reached. For $\sigma < 0, s_0 > s^*$, it is optimal to deplete the soil stock by cultivating the Non-GM variety till the steady state is reached. In figure 1 we depict the case of $\sigma > 0, s_0 < s^*$ and $f^i_{sz_i} > 0, \forall i = 1,2.$ By employing implicit function theorem we obtain $\frac{\partial z_1}{\partial s} = -\frac{H_{sz_1}}{H_{z_1 z_1}} > 0$ which implies that z_1 increases over time until the steady state soil stock is reached. After a certain time period the steady state values of x, z_1, z_2 and s are reached. So the steady state is approached by cultivating a single crop but at the steady state equilibrium both the crops are cultivated. In other words even if the farmer produces either of the two varieties of the crops, steady state is approached most rapidly where the profit maximizing farmer produces an optimal crop mix. So the steady state implies crop diversification and that two with crops having soil conserving features. Therefore this analysis shows a farsighted profit maximizing farmer the way to produce an optimal crop mix as well as the optimal input intensity.

Figure 1: Graphs in (t, x); (t, z_1); (t, z_2) and (t, s) Plane



STABILITY ANALYSIS

Equation (16) and proposition 1 suggests that x is a piecewise constant function and we now analyze the set of differential equations (12) and (13) for a constant x.

For stability analysis we reduce the first order conditions to a pair of differential equations in λ and s, assuming x as a constant. Since $\frac{\partial(H_{z_1}, H_{z_2})}{\partial(z_1, z_2)}$ does not vanish over its entire domain (12) and (13) can be solved globally and uniquely for $z_i = \hat{z}_i(s,\lambda;\beta)$ where $\beta = (p_1, p_2, p_{z_1}, p_{z_2}, M_1, M_2, R_1, R_2, \theta)$.

The elements of the Jacobian matrix J for the system of the equations (12) and (13) are given by $J = \begin{bmatrix} \frac{\partial \dot{s}}{\partial s} & \frac{\partial \dot{s}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial s} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{bmatrix}$ where

$$\frac{\partial \dot{s}}{\partial s} = -h_s^1(s,\hat{z}_1,\psi^1(M_1,\theta R_{c_1}))x - h_s^2(s,\hat{z}_2,\psi^2(M_2,\theta R_{c_2}))(1-x) + G_s(s) - h_{z_1}^1(s,\hat{z}_1,\psi^1(M_1,\theta R_{c_1}))x\frac{\partial \hat{z}_1}{\partial s} - h_{z_2}^2(s,\hat{z}_2,\psi^2(M_2,\theta R_{c_2}))(1-x)\frac{\partial \hat{z}_2}{\partial s} \quad (18)$$

$$\frac{\partial \dot{s}}{\partial \lambda} = -h_{z_1}^1(s,\hat{z}_1,\psi^1(M_1,\theta R_{c_1}))x\frac{\partial \hat{z}_1}{\partial \lambda} - h_{z_2}^2(s,\hat{z}_2,\psi^2(M_2,\theta R_{c_2}))(1-x)\frac{\partial \hat{z}_2}{\partial \lambda} \quad (19)$$

$$\frac{\partial \dot{\lambda}}{\partial s} = -\left[p_1 f_{ss}^1(s,\hat{z}_1,\phi^1(M_1,R_1))\right]x - \left[p_2 f_{ss}^2(s,\hat{z}_2,\phi^2(M_2,R_2))\right](1-x) - p_1 f_{sz_1}^1(s,\hat{z}_1,\phi^1(M_1,R_1))x\frac{\partial \hat{z}_1}{\partial s} - p_2 f_{sz_2}^2(s,\hat{z}_2,\phi^2(M_2,R_2))(1-x)\frac{\partial \hat{z}_2}{\partial s} + \lambda[h_{ss}^1(s,\hat{z}_1,\psi^1(M_1,\theta R_{c_1}))x + h_{ss}^2(s,\hat{z}_2,\psi^2(M_2,\theta R_{c_2}))(1-x) - G_{ss}(s)] + \lambda h_{sz}^1(s,\hat{z}_1,\psi^1(M_1,\theta R_{c_1}))x\frac{\partial \hat{z}_1}{\partial s} + \lambda h_{sz_2}^2(s,\hat{z}_2,\psi^2(M_2,\theta R_{c_2}))(1-x)\frac{\partial \hat{z}_2}{\partial s} \quad (20)$$

$$\frac{\partial\dot{\lambda}}{\partial\lambda}=\delta-p_1 f^1_{sz_1}(s,\hat{z}_1,\phi^1(M_1,R_1))x\frac{\partial\hat{z}_1}{\partial\lambda}-p_2 f^2_{sz_2}(s,\hat{z}_2,\phi^2(M_2,R_2))(1-x)\frac{\partial\hat{z}_2}{\partial\lambda}+$$
$$[h^1_s(s,\hat{z}_1,\psi^1(M_1,\theta R_{c_1}))x+h^2_s(s,\hat{z}_2,\psi^2(M_2,\theta R_{c_2}))(1-x)]+ \tag{21}$$
$$\lambda h^1_{sz}(s,\hat{z}_1,\psi^1(M_1,\theta R_{c_1}))x\frac{\partial\hat{z}_1}{\partial\lambda}+\lambda h^2_{sz_2}(s,\hat{z}_2,\psi^2(M_2,\theta R_{c_2}))(1-x)\frac{\partial\hat{z}_2}{\partial\lambda}-G_s(s)$$

Now the trace of the Jacobian Matrix is given by

$$T=\frac{\partial\dot{s}}{\partial s}+\frac{\partial\dot{\lambda}}{\partial\lambda}$$

Applying implicit function theorem to equation to (10) and (11)

$$\frac{\partial\hat{z}_1}{\partial s}=-\frac{H_{z_1 s}}{H_{z_1 z_1}},\frac{\partial\hat{z}_2}{\partial s}=-\frac{H_{z_2 s}}{H_{z_2 z_2}},\frac{\partial\hat{z}_1}{\partial\lambda}=-\frac{H_{z_1\lambda}}{H_{z_1 z_1}},\frac{\partial\hat{z}_2}{\partial\lambda}=-\frac{H_{z_2\lambda}}{H_{z_2 z_2}}$$

Therefore, T= δ >0.

So, the sum of the Eigen values is positive which means at least one Eigen value is positive. If both the Eigen values are positive then soil stock will grow infinitely large which is not feasible. Thus one Egien value must be negative and that leads us to the following proposition.

Proposition 2: *The equilibrium point of the system of equations (12) and (13) subject to (9)-(11) can be characterized for a constant x by a local saddle point.*

Now as per Goetz (1998) and LaFrance (1992) we impose the restriction of constant soil loss such that

$$h^i_s(s,z_i,\psi^i(M_i,\theta R_{c_1}))\approx G_s(s)\approx 0, i=1,2.$$

Thus the elements of the Jacobian matix become:

$$\frac{\partial \dot{s}}{\partial s} = h^1_{z_1}(s, \hat{z}_1, \psi^1(M_1, \theta R_{c_1}))x \frac{\partial \hat{z}_1}{\partial s} - h^2_{z_2}(s, \hat{z}_2, \psi^2(M_2, \theta R_{c_2}))(1-x)\frac{\partial \hat{z}_2}{\partial s} \quad (18.1)$$

$$\frac{\partial \dot{s}}{\partial \lambda} = -h^1_{z_1}(s, \hat{z}_1, \psi^1(M_1, \theta R_{c_1}))x \frac{\partial \hat{z}_1}{\partial \lambda} - h^2_{z_2}(s, \hat{z}_2, \psi^2(M_2, \theta R_{c_2}))(1-x)\frac{\partial \hat{z}_2}{\partial \lambda} \quad (19.1)$$

$$\begin{aligned}\frac{\partial \dot{\lambda}}{\partial s} &= -\left[p_1 f^1_{ss}(s, \hat{z}_1, \phi^1(M_1, R_1))\right]x - \left[p_2 f^2_{ss}(s, \hat{z}_2, \phi^2(M_2, R_2))\right](1-x) - \\ &p_1 f^1_{sz_1}(s, \hat{z}_1, \phi^1(M_1, R_1))x \frac{\partial \hat{z}_1}{\partial s} - -p_2 f^2_{sz_2}(s, \hat{z}_2, \phi^2(M_2, R_2))(1-x)\frac{\partial \hat{z}_2}{\partial s}\end{aligned} \quad (20.1)$$

$$\frac{\partial \dot{\lambda}}{\partial \lambda} = \delta - p_1 f^1_{sz_1}(s, \hat{z}_1, \phi^1(M_1, R_1))x \frac{\partial \hat{z}_1}{\partial \lambda} - p_2 f^2_{sz_2}(s, \hat{z}_2, \phi^2(M_2, R_2))(1-x)\frac{\partial \hat{z}_2}{\partial \lambda} \quad (21.1)$$

Now solving the equations $H_{z_1} = 0, \forall i = 1,2$ for $z_i = \hat{z}_i(s, \lambda; \beta)$,
i =1,2 we obtain

$$A + B\begin{pmatrix} \frac{\partial \hat{z}_1}{\partial s} & \frac{\partial \hat{z}_2}{\partial \lambda} & \frac{\partial \hat{z}_1}{\partial p_1} & \frac{\partial \hat{z}_1}{\partial p_2} & \frac{\partial \hat{z}_1}{\partial p_{z_1}} & \frac{\partial \hat{z}_1}{\partial p_{z_2}} & \frac{\partial \hat{z}_1}{\partial R_1} & \frac{\partial \hat{z}_1}{\partial R_2} & \frac{\partial \hat{z}_1}{\partial M_1} & \frac{\partial \hat{z}_1}{\partial M_2} & \frac{\partial \hat{z}_1}{\partial \theta} \\ \frac{\partial \hat{z}_2}{\partial s} & \frac{\partial \hat{z}_2}{\partial \lambda} & \frac{\partial \hat{z}_2}{\partial p_1} & \frac{\partial \hat{z}_2}{\partial p_2} & \frac{\partial \hat{z}_2}{\partial p_{z_1}} & \frac{\partial \hat{z}_2}{\partial p_{z_2}} & \frac{\partial \hat{z}_2}{\partial R_1} & \frac{\partial \hat{z}_2}{\partial R_2} & \frac{\partial \hat{z}_2}{\partial M_1} & \frac{\partial \hat{z}_2}{\partial M_2} & \frac{\partial \hat{z}_2}{\partial \theta} \end{pmatrix} = 0 \quad (22)$$

where

B= $\begin{bmatrix} H_{z_1 z_1} & 0 \\ 0 & H_{z_2 z_2} \end{bmatrix}$ and

A= $\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} & A_{19} & A_{110} & A_{111} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} & A_{29} & A_{210} & A_{211} \end{pmatrix}$

The elements of the A matrix are as follows:

$$A_{11} = p_1 f^1_{z_1 s} x, A_{12} = -h^1_{z_1} x, A_{13} = f^1_{z_1} x, A_{14} = 0, A_{15} = -x, A_{16} = 0, A_{17} = p_1 f^1_{z_1\phi^1} \phi^1_{M_1} x - \lambda h^1_{z_1\psi^1} \psi^1_{M_1} x$$

$$A_{18} = 0, A_{19} = p_1 f^1_{z_1\phi^1} \phi^1_{R_1} x, A_{110} = 0, A_{111} = -\lambda h^1_{z_1\psi^1} \psi^1_{\theta} x$$

$$A_{21} = p_2 f^2_{z_2 s}(1-x), A_{22} = -h^2_{z_2}(1-x), A_{23} = 0, A_{24} = f^2_{z_2}(1-x), A_{25} = 0,$$

$$A_{26} = -(1-x), A_{27} = 0, A_{28} = (p_2 f^2_{z_2\phi^2} \phi^2_{M_2} - \lambda h^2_{z_2\psi^2} \psi^2_{M_2})(1-x),$$

$$A_{29} = 0, A_{210} = p_2 f^2_{z_2\phi^2} \phi^2_{R_2}(1-x), A_{211} = -\lambda h^2_{z_2\psi^2} \psi^2_{\theta}(1-x)$$

Solving equation (22) by Cramer's rule we get the following:

$$\frac{\partial \hat{z}_1}{\partial s} => 0, \frac{\partial \hat{z}_2}{\partial s} > 0 \frac{\partial \hat{z}_1}{\partial \lambda} < 0, \frac{\partial \hat{z}_2}{\partial \lambda} < 0, \frac{\partial \hat{z}_1}{\partial p_1} > 0, \frac{\partial \hat{z}_2}{\partial p_1} = 0, \frac{\partial \hat{z}_1}{\partial p_2} = 0, \frac{\partial \hat{z}_2}{\partial p_1} > 0, \frac{\partial \hat{z}_1}{\partial p_{z_1}} < 0,$$

$$\therefore \frac{\partial \hat{z}_2}{\partial p_{z_1}} = 0, \frac{\partial \hat{z}_1}{\partial p_{z_2}} = 0, \frac{\partial \hat{z}_2}{\partial p_{z_2}} < 0, \frac{\partial \hat{z}_1}{\partial M_1} > 0, \frac{\partial \hat{z}_2}{\partial M_1} = 0, \frac{\partial \hat{z}_1}{\partial M_2} = 0, \frac{\partial \hat{z}_2}{\partial M_2} > 0, \frac{\partial \hat{z}_1}{\partial R_1} > 0, \quad (23)$$

$$\frac{\partial \hat{z}_2}{\partial R_1} = 0, \frac{\partial \hat{z}_1}{\partial R_2} = 0, \frac{\partial \hat{z}_2}{\partial R_2} > 0, \frac{\partial \hat{z}_1}{\partial \theta} > 0, \frac{\partial \hat{z}_2}{\partial \theta} > 0.$$

Note that here the restrictions used are

$$f^i_{sz_i} > 0, f^i_{z_i\phi^i} >, h^i_{z_i\psi^i} < 0.$$

Using (23) we find

$$\frac{\partial \dot{s}}{\partial s} < 0, \frac{\partial \dot{s}}{\partial \lambda} > 0, \frac{\partial \dot{\lambda}}{\partial s} >=< 0, \frac{\partial \dot{\lambda}}{\partial \lambda} > 0.$$

Now the slope of the two isoclines $\dot{s} = 0$ and $\dot{\lambda} = 0$ can be obtained by implicit function theorem:

$$\frac{\partial \lambda}{\partial s}\Big|\dot{s}=0=\frac{\partial \dot{s}/\partial s}{\partial \dot{s}/\partial \lambda},\ \frac{\partial \lambda}{\partial s}\Big|\dot{\lambda}=0=\frac{\partial \dot{\lambda}/\partial s}{\partial \dot{\lambda}/\partial \lambda} \qquad (24)$$

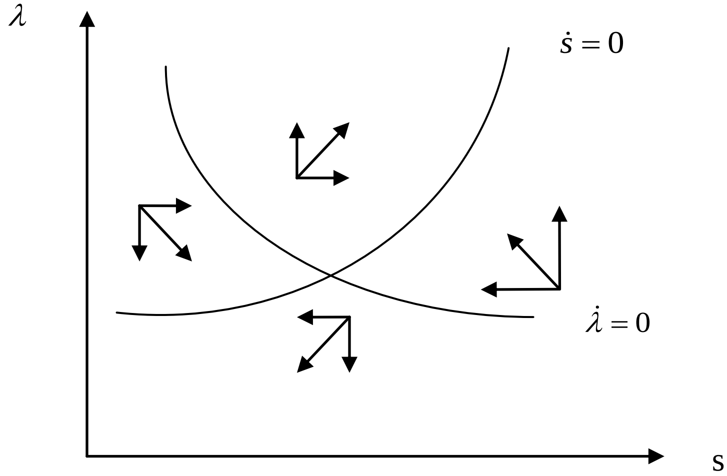
Using (24) we can say that the slope of the isocline $\dot{s}=0$ is positive where as slope of the isocline $\dot{\lambda}=0$ is positive if $\sum_{i=1}^{2} p_i f_{ss}^{i} \eta_i$ is small in absolute value where $\eta_1 = x, \eta_2 = 1-x$ and negative if $\sum_{i=1}^{2} p_i f_{ss}^{i} \eta_i$ is large in absolute value.

Thus we have the following two cases:

Case 1: When both $\dot{s}=0$ and $\dot{\lambda}=0$ are positively sloped. It requires $f_{sz_i}^{i} > 0$ and $\sum_{i=1}^{2} p_i f_{ss}^{i} \eta_i$ to be small in absolute value. If we draw the phase diagram in the state-costate phase plane we find an unstable equilibrium which is not a saddle point.

Case 2: When $\dot{s}=0$ is positively sloped and $\dot{\lambda}=0$ is negatively sloped. It requires $f_{sz_i}^{i} > 0$ and $\sum_{i=1}^{2} p_i f_{ss}^{i} \eta_i$ to be large in absolute value. If we draw the phase diagram in the state-costate phase plane we find perfect saddle point equilibrium.

Figure 2: Phase Diagram: Saddle Point Equilibrium



Here two streamlines are converging towards the equilibrium whereas the other two are diverging away from the equilibrium, thereby giving the perfect saddle point equilibrium.

POLICY ANALYSES

Now we analyze the impact of variations in the parameters on the steady state values given, $z_i > 0$, i=1,2 and $x \in (0,1)$. It allows us to formulate policies which will promote the cultivation of a combination of both GM and Non-GM varieties of the same crop to help soil conservation.
The system of equation to be considered is:

$$H_x = 0$$
$$\dot{s} \equiv H_\lambda = 0 \qquad (25)$$
$$\dot{\lambda} \equiv \delta\lambda - H_s = 0$$

The Jacobian matrix of this system of equations is given by

$$J = \begin{pmatrix} H_{xx} & H_{x\lambda} & H_{xs} \\ H_{\lambda x} & H_{\lambda\lambda} & H_{\lambda s} \\ -H_{sx} & \delta & -H_{ss} \end{pmatrix}$$

$H_{xx} = 0, H_{x\lambda} = -(h^1 - h^2), H_{xs} = p_1 f_s^i - p_2 f_s^2, H_{\lambda x} = -(h^1 - h^2), H_{\lambda\lambda} = 0, H_{\lambda s} = 0,$
$-H_{ss} = -p_1 f_{ss}^1 x - p_2 f_{ss}^2 (1-x)$

Now we assume that $h^2 > h^1$ and following Goetz it can be shown that $H_{xs} < 0$ provided $f_{sz_i}^i > 0$ which is already assumed by us. Thus we get

$$|J| = H_{\lambda x}(\delta H_{xs} + H_{\lambda x} H_{ss}) < 0 \tag{26}$$

Moreover we substitute $z_i = \hat{z}_i(s, \lambda; \beta)$, i=1,2. Thus we are able to determine the impact of a change in the parameters on the state and costate variables of the steady state equilibrium and also the steady state equilibrium value of the control variable x. Since $|J| < 0,$ when evaluated at the steady state, (9),(12) and (13) can be solved uniquely for $x(\beta_1), \lambda(\beta_1)$ and $s(\beta_1)$, where $\beta_1 = (\beta, \delta)$.Hence the implicit function theorem yields

$$C + J \begin{pmatrix} \frac{\partial x^*}{\partial p_1} & \frac{\partial x^*}{\partial p_2} & \frac{\partial x^*}{\partial p_{z_1}} & \frac{\partial x^*}{\partial p_{z_2}} & \frac{\partial x^*}{\partial R_1} & \frac{\partial x^*}{\partial R_2} & \frac{\partial x^*}{\partial M_1} & \frac{\partial x^*}{\partial M_2} & \frac{\partial x^*}{\partial \theta} & \frac{\partial x^*}{\partial \delta} \\ \frac{\partial \lambda^*}{\partial p_1} & \frac{\partial \lambda^*}{\partial p_2} & \frac{\partial \lambda^*}{\partial p_{z_1}} & \frac{\partial \lambda^*}{\partial p_{z_2}} & \frac{\partial \lambda^*}{\partial R_1} & \frac{\partial \lambda^*}{\partial R_2} & \frac{\partial \lambda^*}{\partial M_1} & \frac{\partial \lambda^*}{\partial M_2} & \frac{\partial \lambda^*}{\partial \theta} & \frac{\partial \lambda^*}{\partial \delta} \\ \frac{\partial s^*}{\partial p_1} & \frac{\partial s^*}{\partial p_2} & \frac{\partial s^*}{\partial p_{z_1}} & \frac{\partial s^*}{\partial p_{z_2}} & \frac{\partial s^*}{\partial R_1} & \frac{\partial s^*}{\partial R_2} & \frac{\partial s^*}{\partial M_1} & \frac{\partial s^*}{\partial M_2} & \frac{\partial s^*}{\partial \theta} & \frac{\partial s^*}{\partial \theta} \end{pmatrix} = 0 \tag{27}$$

where

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & C_{17} & C_{18} & C_{19} & C_{110} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} & C_{27} & C_{28} & C_{29} & C_{210} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} & C_{37} & C_{38} & C_{39} & C_{310} \end{pmatrix}$$

and

$$C_{11} = f^1, C_{12} = f^2, C_{13} = -z_1, C_{14} = z_2,$$

$$C_{15} = p_1 f^1_{\phi^1} \phi^1_{R_1}, C_{16} = -p_2 f^2_{\phi^2} \phi^2_{R_2}, C_{17} = p_1 f^1_{\phi^1} \phi^1_{M_1} - \lambda h^1_{\psi^1} \psi^1_{M_1},$$

$$C_{18} = -(p_2 f^2_{\phi^2} \phi^2_{M_2} - \lambda h^2_{\psi^2} \psi^2_{M_2}), C_{19} = -\lambda\left(h^1_{\psi^1} \psi^1_\theta - h^2_{\psi^2} \psi^2_\theta\right), C_{110} = 0,$$

$$C_{21} = -h_{z_1} x \frac{\partial \hat{z}_1}{\partial p_1}, C_{22} = -h^2_{z_2} (1-x) \frac{\partial \hat{z}_2}{\partial p_2},$$

$$C_{23} = -h^1_{z_1} x \frac{\partial \hat{z}_1}{\partial p_{z_1}}, C_{24} = -h^2_{z_2} (1-x) \frac{\partial \hat{z}_2}{\partial p_{z_2}}, C_{25} = -h^1_{z_1} x \frac{\partial \hat{z}_1}{\partial R_1},$$

$$C_{26} = -h^2_{z_2} (1-x) \frac{\partial \hat{z}_2}{\partial R_2}, C_{27} = -h^1_{z_1} x \frac{\partial \hat{z}_1}{\partial M_1} - h^1_{\psi^1} \psi^1_{M_1} x,$$

$$C_{28} = -h^2_{z_2} (1-x) \frac{\partial \hat{z}_2}{\partial M_2} - h^2_{\psi^2} \psi^2_M \ (1-x),$$

$$C_{29} = -h^1_{z_1} x \frac{\partial \hat{z}_2}{\partial \theta} - h^1_{\psi^1} \psi^1_\theta x - h^2_{z_2} (1-x) \frac{\partial \hat{z}_2}{\partial \theta} - h^2_{\psi^2} \psi^2_\theta (1-x), C_{30} = 0$$

$$C_{31} = -f^1_s x - p_1 f^1_{sz_1} \frac{\partial \hat{z}_1}{\partial p_1} x, C_{32} = -f^2_s (1-x) - p_2 f^2_{sz_2} (1-x) \frac{\partial \hat{z}_2}{\partial p_2},$$

$$C_{33} = -p_1 f^1_{sz_1} x \frac{\partial z_1}{\partial p_{z_1}}, C_{34} = -p_2 f^2_{sz_2} (1-x) \frac{\partial \hat{z}_2}{\partial p_{z_2}}, C_{35} = -p_1 f^1_{sz_1} x \frac{\partial \hat{z}_1}{\partial R_1} - p_1 f^1_{s\phi^1} \phi^1_{R_1} x,$$

$$C_{36} = -p_2 f^2_{sz_2} (1-x) \frac{\partial \hat{z}_2}{\partial R_2} - p_2 f^2_{s\phi^2} \phi^2_{R_2} (1-x), C_{37} = -p_1 f^1_{sz_1} x \frac{\partial \hat{z}_1}{\partial M_1} - p_1 f^1_{sz_1} x \phi^1_{M_1},$$

$$C_{38} = -p_2 f^2_{sz_2}(1-x)\frac{\partial \hat{z}_2}{\partial M_2} - p_2 f^2_{s\phi^2}\phi^2_{M_2}(1-x),$$

$$C_{39} = -p_1 f^1_{sz_1}\frac{\partial \hat{z}_1}{\partial \theta}x - p_2 f^2_{sz_2}\frac{\partial \hat{z}_2}{\partial \theta}(1-x), C_{310} = \lambda \tag{28}$$

Proposition 3: *Improvement in the capacity of root characteristic of the crop to anchor the soil (θ) leads to an increment in the equilibrium soil stock and a fall in the equilibrium value of the percentage of land devoted to GM crop, assuming that this feature of root characteristic of GM variety has a much greater role in preventing soil erosion compared to its Non-GM counterpart.*

$$\frac{\partial s^*}{\partial \theta} = \frac{\begin{array}{c}(h^2 - h^1)[(p_2 f^2_s - p_1 f^1_s)(h^1_{z_1} x \,\partial \hat{z}_1/\partial \theta + h^1_\theta \psi^1_\theta x + h^2_{z_2}(1-x)\,\partial \hat{z}_2/\partial \theta + h^2_{\psi^2}\psi^2_\theta) - \\ (h^2 - h^1)(p_1 f^1_{sz_1} x \,\partial \hat{z}_1/\partial \theta + p_2 f^2_{sz_2}\,\partial \hat{z}_2/\partial \theta (1-x))] + \lambda[h^1_{\psi^1}\psi^1_\theta - h^2_{\psi^2}\psi^2_\theta]\delta(h^2 - h^1)\end{array}}{|J|} \tag{29}$$

$$\frac{\partial x^*}{\partial \theta} = \frac{\begin{array}{c}(h^2 - h^1)[(h^1_{z_1} x \,\partial \hat{z}_1/\partial \theta + h^1_{\psi^1}\psi^1_\theta x + h^2_{z_2}(1-x)\,\partial \hat{z}_\theta/\partial \theta + h^2_{\psi^2}\psi^2_\theta(1-x)](p_1 f^1_{ss} x + p_2 f^2_{ss}(1-x))] + \\ (p_1 f^1_s - p_2 f^2_s)(h^1_{z_1} x \,\partial \hat{z}_1/\partial \theta + h^1_{\psi^1}\psi^1_\theta x + h^2_{z_2}(1-x)\,\partial \hat{z}_2/\partial \theta + h^2_{\psi^2}\psi^2_\theta(1-x))\delta\end{array}}{|J|} \tag{30}$$

Now we assume that $\left|h^1_{\psi^1}\psi^1_\theta\right| > \left|h^2_{\psi^2}\psi^2_\theta\right|$ ie the impact of that feature of the root which helps to anchor the soil has a greater influence on preventing soil erosion in case of the GM variety rather that the Non-GM one. Again, $\left|h^1_{\psi^1}\psi^1_\theta x\right| > \left|h^1_{z_1} x \,\partial \hat{z}_1/\partial \theta\right|$ such that fall in soil erosion due to the specific root characteristic of GM crop is more than the increase in soil erosion due to enhanced input usage. This is just opposite in case of the Non-GM variety so that we can use the restriction $\left| h^3_{\psi^2} \psi^2_\theta (1-x) \right| < \left| h^2_{z_2} (1-x) \partial \hat{z}_2 / \partial \theta \right|$. Moreover it can well be assumed that the resultant erosion of soil in Non-GM cultivated land is greater than that of GM cultivated land in absolute sense such that $\left| h^1_{z_1} x \partial \hat{z}_1 / \partial \theta + h^1_{\psi^1} \psi^1_\theta x \right| > \left| h^2_{z_2} (1-x) \partial \hat{z}_2 / \partial \theta + h^2_{\psi^2} \psi^2_\theta (1-x) \right|$. Given all the above conditions we can conclude that with improvement in the soil-anchoring characteristic of the root there will be accretion of equilibrium soil stock. By the same logic stated in the last proposition we can justify the fall in equilibrium value of the percentage of land devoted to GM cultivation.

Proposition 4: *Increase in organic matter in the topsoil of the land under GM cultivation leads to an increase in steady state value of the soil stock but a fall in the equilibrium value of the percentage of land devoted to GM crop, both under the same restriction.*

$$\frac{\partial s^*}{\partial M_1} = \frac{\begin{array}{l} \left(h^2 - h^1\right)\left[\left(p_2 f^2_s - p_1 f^1_s\right)\left(h^1_{z_1} x \partial \hat{z}_1 / \partial M_1 + h^1_{\psi} \psi^1_{M_1} x\right) - \\ \left(h^2 - h^1\right)\left(p_1 f^1_{sz_1} x \partial \hat{z}_1 / \partial M_1 + p_1 f^1_{s\phi^1} \phi^1_{R_1} x\right) + \left[-p_1 f^1_{\phi^1} \phi^1_{M_1} + \lambda h^1_{\psi^1} \psi^1_{M_1}\right] \delta \left(h^2 - h^1\right) z_1 \end{array}}{|J|} \quad (31)$$

$$\frac{\partial x^*}{\partial M_1} = \frac{\begin{array}{l} \left(h^2 - h^1\right)\left[\left(h^1_{z_1} x \partial \hat{z}_1 / \partial M_1 + h^1_{\psi^1} \psi^1_{M_1} x\right)\left(p_1 f^1_{ss} x + p_2 f^2_{ss} (1-x)\right)\right] \\ + \left(p_1 f^1_s - p_2 f^2_s\right)\left(h^1_{z_1} x \partial \hat{z}_1 / \partial M_1 + h^1_{\psi^1} \psi^1_{M_1} x\right) \delta \end{array}}{|J|} \quad (32)$$

Here we assume that with the presence of organic matter in the topsoil of GM cultivated land fall in soil erosion dominates the aggravation of erosion due to input use with intensification of cultivation, in absolute value, such that the following restriction can be utilized: $\left|h^1_{\psi^1}\psi^1_{M_1}x\right| > \left|h^1_{z_1}x \,{\partial \hat{z}_1}/{\partial M_1}\right|$.This leads us to a positive relation between equilibrium soil stock and the organic matter in the top soil in GM cultivated land. Thus conservation tillage plays an important role in accumulating the organic matter in the top soil such that even if input use increases with production process, the soil gets less eroded leading to an increment in soil stock in overall analysis. However, with increase in organic matter in topsoil, there is a fall in the steady state value of percentage of land devoted to GM cultivation. This can be justified in the following way. With cultivation of GM crop as the soil gets less eroded and there is limit beyond which accumulation of soil does not make any change in the magnitude of soil loss, the targeted level of soil depth at the steady state can be achieved by cultivating a lesser amount of GM variety.

Proposition 5: *Increase in the R&D investment in GM cultivation will lead to an increment in the Steady State value of the percentage of the land devoted to GM cultivation whereas the effect on equilibrium soil stock is ambiguous*.

$$\frac{\partial x^*}{\partial R_1} = \frac{\left(h^2 - h^1\right)\left[h^1_{z_1} x \,{\partial \hat{z}_1}/{\partial R_1}\left(p_1 f^1_{ss} x + p_2 f^2_{ss}(1-x)\right)\right] + \left(p_1 f^1_s - p_2 f^2_s\right) h^1_{z_1} x \,{\partial \hat{z}_1}/{\partial R_1}\,\delta}{|J|} > 0 \quad (33)$$

$$\frac{\partial s^*}{\partial R_1} = \frac{\begin{array}{l}\left(h^2 - h^1\right)\left[h^1_{z_1}\left(p_2 f^2_s - p_1 f^1_s\right) x \,{\partial \hat{z}_1}/{\partial R_1} - \\ \left(h^2 - h^1\right)\left(p_1 f^1_{sz_1}\,{\partial \hat{z}_1}/{\partial R_1} + p_1 f^1_{s\phi^1}\phi^1_{R_1} x\right] - p_1 f^1_{\phi^1}\phi^1_{R_1}\left(h^2 - h^1\right) z_1 \delta\end{array}}{|J|} \quad (34)$$

Given the sign restrictions used earlier, equation (33) suggests that R&D investment in GM cultivation and Steady state value of the percentage of land devoted to GM variety are positively related. Thus with the Government policy of enhancement of R&D investment in GM sector there will be new varieties of GM crop available which have the property of longer root that helps to tighten the soil. With the dissemination of this knowledge and availability of those seeds developed in the laboratories farmers will be influenced to cultivate more of the GM variety. However we can not predict unambiguously about the effect on the equilibrium soil stock.

Proposition 6: *Imposition of subsidy on the inputs used in GM variety of the crop leads to increase in steady state value of the percentage of land devoted to GM variety, though the effect on the steady state soil stock is ambiguous.*

Solving equation (27) by Cramer's rule we get

$$\frac{\partial x^*}{\partial p_{z_1}} = \frac{\left(h^2 - h^1\right) h^1_{z_1} x \, {\partial \hat{z}_1}/{\partial p_{z_1}} \left(p_1 f^1_{ss} x + p_2 f^2_{ss}(1-x)\right) + \left(p_1 f^1_s - p_2 f^2_s\right) h^1_{z_1} x \, {\partial \hat{z}_1}/{\partial p_{z_1}} \, \delta}{|J|} < 0 \quad (35)$$

$$\frac{\partial s^*}{\partial p_{z_1}} = \frac{\left(h^2 - h^1\right)[h^1_{z_1}\left(p_2 f^2_s - p_1 f^1_s\right) - \left(h^2 - h^1\right) p_1 f^1_{sz_1}] x \, {\partial \hat{z}_1}/{\partial p_{z_1}} + \left(h^2 - h^1\right) z_1 \delta}{|J|} \quad (36)$$

Given all the sign restrictions earlier it can be concluded that there is a negative relation between the price of the inputs to be used in GM cultivation and the equilibrium value of the percentage of the land devoted to GM variety. It implies that if the Government imposes a subsidy on the input price of GM, there will be a fall in the price of those inputs and the farmers will be attracted to devote more land to GM crop. As far as the equilibrium value of the soil stock is concerned we can not conclude anything unambiguously.

CONCLUSION

This paper addresses the important issue of the erosion of a renewable resource like soil. In a dynamic optimization framework, it is possible to show that, at the long run equilibrium steady state farmers cultivate an optimal combination of a Non- GM and a GM crop, though the steady state is approached most rapidly by producing a single crop. The present paper highlights the role of GM crops having longer, branched and voluminous roots that will provide anchorage, thereby binding soil particles and organic matter to control soil erosion. Our model postulates that the improvement in the capacity to anchor the soil particles by plants will raise the long run steady state soil stock under certain conditions**.** In fact there are evidences of coexistence of GM and non GM crops side by side in different parts of the world. Thus utilization of positive role of GM crops will provide a sense of new insight into the problem of soil erosion. Moreover, the role of R&D investment in agriculture in this model was incorporated which was absent in earlier literatures on soil erosion. A significant observation can be made here that the two parameters i.e. the percentage of organic matter in the top soil and a specific root characteristic, involved in the erosion function have played positive role in the prevention of soil erosion and that with the assumption of GM crops are being able to take greater advantage in conservation tillage by their roots. The R&D investment and the policy towards subsidizing inputs are leading to an increment in area under GM cultivation though their effect on long run soil stock is uncertain. However, our expectation is that the R&D investment can be made a function of the root character such that the investment is made in developing that particular feature and introduce that in the erosion function, then a more defining role of the R&D investment can be obtained in incrementing the soil depth. We propose this as a future extension of the present model. Another important aspect needs to be addressed i.e. the issue of uncertainly involved in GM crop production. Uncertainty is a major issue in agriculture and is expected is play even a bigger role in the situation where a new technology (GM cultivation) is introduced. As this is a baseline model, this kind of complications can well be incorporated in future to make it more realistic.

APPENDIX

Proof of proposition 1

We start by assuming that a singular path exists where x lies in the interior of [0, 1]. Now, $\sigma = 0 \Rightarrow$

$$\left[p_1 f^1(s, z_1, \phi^1(M_1, R_1)) - p_{z_1} z_1\right] - \left[p_2 f^2(s, z_2, \phi^2(M_2, R_2)) - p_{z_2} z_2\right] - \lambda[h^1(s, z_1, \psi^1(M_1, \theta R_{c_1})) - h^2(s, z_2, \psi^2(M_2, \theta R_{c_2}))] = 0$$

$$\Rightarrow \lambda = \frac{\left(p_1 f^1 - p_{z_1} z_1\right) - \left(p_2 f^2 - p_{z_2} z_2\right)}{h^1 - h^2} \qquad \text{(A1)}$$

Taking time derivative we get

$$\dot{\lambda} = \frac{\left(p_1 f^1_{z_1} - p_{z_1} - \lambda h^1_{z_1}\right)\dot{z}_1 - \left(p_2 f^2_{z_2} - p_{z_2} - \lambda h^2_{z_2}\right)\dot{z}_2 + \left(p_1 f^1_s - p_2 f^2_s - \lambda\left(h^1_s - h^2_s\right)\right)\dot{s}}{h^1 - h^2} \qquad \text{(A2)}$$

From equation (12) and (13), the first two parentheses of the numerator sum to zero. Now, we equate (A2) and (15) along with utilizing (A1) for λ and (14) for $\dot{s}$.

$$\delta\lambda - p_1 f^1_s x - p_2 f^2_s (1-x) + \lambda h^1_s x + \lambda h^2_s (1-x) - \lambda G_s = \frac{\left(p_1 f^1_s - p_2 f^2_s - \lambda\left(h^1_s - h^2_s\right)\right)\left(-h^1 x + h^2(1-x) + G(s)\right)}{h^1 - h^2}$$

After some calculations x gets cancelled out and we get the following result:

$$\left(\frac{h^2-G}{h^1-h^2}\right)\left[p_1 f_s^1 - p_2 f_s^2 - \left(\frac{p_1 f^1 - p_{z_1} z_1 - p_2 f^2 + p_{z_1} z_2}{h^1-h^2}\right)\left(h_s^1 - h_s^1\right)\right] +$$
$$\left(\frac{p_1 f^1 - p_{z_1} z_1 - p_2 f^2 + p_{z_1} z_2}{h^1-h^2}\right)\left(\delta - G_s + h_s^2\right) - p_2 f_s^2 = 0$$

(A3)

Some more algebraic manipulations and calculations lead to:

$$\left(\frac{h^2-G}{h^2-h^1}\right)p_1 f_s^1 + p_2 f_s^2\left(\frac{G-h^1}{h^2-h^1}\right) = \lambda\left(\delta + \frac{h_s^1(h^2-G)}{(h^2-h^1)} + h_s^2\frac{\left(G-h^1\right)}{(h^2-h^1)} - G_s\right)$$

(A4)

To find the condition where equation (A4) is satisfied consider $\dot{\lambda} = 0$. Comparing equation (13) and (A3) we get,

$$x = \left(\frac{h^2-G}{h^2-h^1}\right), (1-x) = \left(\frac{G-h^1}{h^2-h^1}\right) \quad \text{(A5)}$$

Thus equation (A4) provides equation (17) of proposition 1.Now, equation (A2) implies when $\dot{s} > 0$, $\dot{\lambda} = 0$. Hence the singular path coincides with steady state.

Now, let us assume there exist such singular path which is not identical to steady state and therefore it must hold that

$$\dot{\sigma} = H_{xz_1}\dot{z}_1 + H_{xz_1}\dot{z}_2 + H_{xs}\dot{s} + H_{x\lambda}\dot{\lambda} = 0 = H_{xs}\dot{s} + H_{x\lambda}\dot{\lambda} = 0 \quad \text{(A6)}$$

since $H_{xz_t} = 0, \forall 1,2$ from (12) and (13).

For any singular path off the steady state, $\dot{\lambda} \neq \dot{s} \neq 0$. Thus for the existence of singular path off the steady state the sufficient condition is $H_{xs} = H_{x\lambda} = 0$.

Now,
$$H_{xs} = p_1 f_s^1 - p_2 f_s^2 - \lambda(h_s^1 - h_s^2) \neq 0$$
$$H_{x\lambda} = -(h^1 - h^2) \neq 0, \because h^1 \neq h^2$$

Therefore, a singular path is identical to a steady state provided that for any time interval of positive length the following condition obtained from (A6) does not hold:

$$H_{xs} \Big/ H_{x\lambda} = -\dot{\lambda} \Big/ \dot{s}$$

$$\Rightarrow \frac{\left(p_2 f_s^1 - \lambda h_s^1\right) - \left(p_1 f_s^1 - \lambda h_s^1\right)}{h^2 - h^1} = \frac{\dot{\lambda}}{\dot{s}}$$

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